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M.L. Estimator for Fuzzy Survival Function to the Kidney Failure Patients

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***Abstract*—The dependence of traditional statistical methods in finding estimators and testing hypotheses depends on data that take specific values. However, uncertainty (Fuzzy) appears in most data, including survival time data, which requires researchers to apply Fuzzy methods in general when finding estimators, particularly the estimators of a survival function. In this research, the experimental method (simulation) was used to compare Fuzzy methods (definition of fuzzy logic, cut level -δ, Buckley) at three fuzzy degrees (0, 0.5, 1), for selected survival times (t = 2, 4, 6, 8, 10, 12) in days. To determine the best fuzzy method used to find the Maximum Likelihood Estimators of the fuzzy survival function depends on the Avery Mean Squares Error (AMES) for the lower and upper bound estimators of the fuzzy survival function for each method. From the experimental method, we came to an advantage of defining fuzzy logic for the maximum likelihood estimator for the survival function of Weibull distribution (distribution of survival times data in the research). The maximum likelihood estimator calculated according to the method of defining fuzzy logic for patients' survival times with kidney failure collected from sections of the dialysis department in educational hospitals (Baghdad, Al-Kindi, Al-Karama, Al-Kadhimiya). This research found that the probability of fuzzy survival for two days to the patients with kidney failure reaches 0.7504 and this probability decreases with an increase in the time of stay until it reaches 0.07704 the probability of staying for a month.**

***Keywords*—fuzzy logic; survival times; fuzzy survival function; cut level -δ; buckley.**

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I. INTRODUCTION

The superiority and progress achieved by the classical statistical approach in studying, analyzing, and interpreting the results for different phenomena depends on the random principle. This approach remains limited to taking into account the various sources of uncertainty in the data. The fuzzy is one type of uncertainty in the phenomenological data to overcome the shortcoming (deficiency). The fuzzy statistical approach in this research is an analysis phenomenon under uncertain circumstances. The Azerbaijani scientist Zadeh in 1965 developed the concept of fuzzy groups theory where each element belongs to the fuzzy groups according to a specific membership [1], [2]. Survival functions are the most essential and well-known statistical functions that are widely used to estimate the survival probability for people suffering from specific diseases. The recorded survival times for a sample of patients [3] has been known that it suffers from uncertainty due to the inaccuracy of the patient's information about the contracted date of the disease or the hospital review, as well as the lack of or errors due to unstable conditions in Iraq [4]. Accordingly; the idea of research has evolved depends on the uncertainty of the survival times. In general, to determine the best fuzzy approach, it is required to find the maximum likelihood estimator for the survival function. It can be applied to estimate the fuzzy survival function of kidney failure disease which has become a worrying disease because it is dangerous for human life. An Australian study revealed that about 2.3 million patients with kidney failure in the world are died each year and it is predicted that the number of patients with kidney failure will arise around the world to 5.4 million people by 2030 [5], [6].

The multiplicity of methods is for adopting the fuzzy principle in finding statistical estimators in various estimation methods (parametric, non-parametric). Inability to prove the advantage of one method over another with mathematical derivation leads to difficulty to prefer one of these methods except by using an experimental method. This led to a problem finding the best maximum likelihood estimator to the fuzzy survival function for patients with kidney failure to solve a real main problem in the dialysis process for patients in dialysis centers [7].

The limitation of the research covers two dimensions. The first dimension includes methods used to adopt fuzzy in the survival times. The second dimension is setting the scheduled dialysis process according to the priorities of the cases. It is easy to crystallize the research's primary goal in calculating the fuzzy probability for patients at every degree of fuzzy determined in the data ($0 < \delta < 1$). It can help the specialized doctors schedule dialysis times for patients based on prioritizing critical cases that are less likely to survive.

II. MATERIALS AND METHOD

A. Maximum likelihood Estimation for Fuzzy Survival Function

This section describes the method of obtaining the maximum likelihood estimators for survival times (t_i). It is assumed that the Weibull distribution with two parameters follows the probability density function [8]:

$$f(t|\lambda,\beta) = \frac{\beta}{\lambda}\left(\frac{t}{\lambda}\right)^{\beta-1} exp\left[-\left(\frac{t}{\lambda}\right)^{\beta}\right], \quad I(0,\infty)^{t}, \lambda > 0, \beta > 0 \quad (1)$$

the maximum likelihood function will be:

$$L(t_i, \lambda, \beta) = \prod_{i=1}^{n} f(t_i, \lambda, \beta) \quad (2)$$

where $t_1, < \cdots, < t_n$ represents the survival times for a sample with size. $t_1, < \ldots\ldots, < t_r$ represents the observed survival times (occurrence of death event). $t_n - t_r = t_q$, $q = r+1, \ldots, n$ represents the censored survival times (no event of death). Thus, the function of the maximum of likelihood distribution of Weibull can be written as shown below [9]:

$$L(\lambda, \beta; t_i) = \prod_{i=1}^{r} f(t_i) \prod_{i=r+1}^{n} S(t_i) \quad (3)$$

Where: $f(t_i)$: probability density function of Weibull distribution for observed survival times ($i = 1,2,\ldots,r$). $S(t_i)$: survival function of Weibull distribution for censored survival times ($i = r+1, \ldots, n$). Depending on the formula of the risk function, the maximum likelihood function is as follows [10]:

$$L(\lambda, \beta; t_i) = \prod_{i=1}^{r} h(t_i)\, S(t_i) \prod_{i=r+1}^{n} S(t_i) \quad (4)$$

By substituting the risk function and the survival function for the Weibull distribution in equation (4), we obtain:

$$L(\lambda, \beta; t_i) = \prod_{i=1}^{r}\left[\frac{\beta}{\lambda}\left(\frac{t_i}{\lambda}\right)^{\beta-1} exp\left[-\left(\frac{t_i}{\lambda}\right)^{\beta}\right]\right] \prod_{i=r+1}^{n}\left[exp\left[-\left(\frac{t_i}{\lambda}\right)^{\beta}\right]\right] \quad (5)$$

By partially deriving the maximum likelihood function concerning the two parameters of the Weibull distribution (λ, β) and equality the derivative to zero, we obtained the formulas for calculating the estimators of the maximum likelihood according to the following [11]:

$$\hat{\lambda} = \left[\frac{1}{r}\left(\sum_{i=1}^{n} (t_i)^{\hat{\beta}}\right)\right]^{1/\hat{\beta}} \quad (6)$$

$$\frac{1}{\hat{\beta}} = \frac{\sum_{i=1}^{n} t_i^{\hat{\beta}} ln(t_i)}{\sum_{i=1}^{n} t_i^{\hat{\beta}}} - \frac{1}{r}\sum_{i=1}^{r} ln(t_i) \quad (7)$$

Using the Newton-Ravison method, we obtained the maximum likelihood estimator of the parameter β. It was substituted in equation (6) to obtain the maximum likelihood estimator of the parameter λ. By substituting he maximum likelihood estimators in the formula of the survival function below [12]:

$$S_T(t) = exp\left[-\left(\frac{t}{\lambda}\right)\right] \quad (8)$$

we obtained maximum likelihood estimator of the survival function according to the following:

$$\hat{S}(t) = exp\left[-\left(\frac{t}{\hat{\lambda}}\right)^{\hat{\beta}}\right] \quad (9)$$

Depending on the maximum likelihood estimator of the survival function equation (9), the fuzzy survival function estimator was calculated according to the equation's fuzzy methods (10). The Fuzzy Logic Definition approach is based on the concept of fuzzy logic established by the scientist Zadeh [13]. The membership function for fuzzy numbers multiply with fuzzy probability is equal to the membership function's expected value.

$$P(\tilde{A}) = \int m_{\tilde{A}}(x)dP \quad (10)$$

If $\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n$ is a fuzzy number that represents the fuzzy survival duration of the survival data in size (n) of the observations and that $m_{\tilde{t}_1}(.), m_{\tilde{t}_2}(.), \ldots, m_{\tilde{t}_n}(n)$ membership Functions for each fuzzy number. The Weibull distribution function for the fuzzy variable $\tilde{t}$ will be as follows [14]:

$$f(\tilde{t}; \lambda, \beta) = \int f(t; \lambda, \beta)\, m_{\tilde{t}}(t)dt \quad (11)$$

Assuming that the fuzzy numbers $\tilde{t}_i, i = 1, \ldots, n$ are random fuzzy independent variables with similar distribution (i i d). The assumption that the membership functions of these numbers are independent. On this basis and by formula (11) the maximum likelihood function of the fuzzy variable $\tilde{t}$ as follows [15]:

$$L(\lambda, \beta; \tilde{t}_i) = \prod_{i=1}^{n} \int f(t; \lambda, \beta)\, m_{\tilde{t}_i}(t)dt \quad (12)$$

Depending on formula (5), the maximum likelihood function can be calculated using the formula (12) as follows:

$$L(\lambda, \beta; \tilde{t}_i) = \prod_{i=1}^{r} \int \left(\frac{\beta}{\lambda}\right)\left(\frac{t}{\lambda}\right)^{\beta-1} \quad (13)$$

Taking the natural logarithms and partial derivative of equation (13) we get:

$$LnL(\lambda, \beta; \tilde{t}_i) = rLn\left(\frac{\beta}{\lambda}\right) + \sum_{i=1}^{r} Ln \int \left(\frac{t}{\lambda}\right)^{\beta-1} m_{\tilde{t}_i}(t)dt \quad (14)$$

$$\frac{dLnL(\lambda, \beta; \tilde{t}_i)}{d\lambda} = \frac{-r}{\lambda} - \sum_{i=1}^{r}\left[\frac{\int\left(\frac{\beta-1}{\lambda}\right)\left(\frac{t}{\lambda}\right)^{\beta-1} m_{\tilde{t}_i}(t)dt}{\int\left(\frac{t}{\lambda}\right)^{\beta-1} m_{\tilde{t}_i}(t)dt}\right] + \sum_{i=1}^{n}\left[\frac{\int\left(\frac{\beta}{\lambda}\right)\left(\frac{t}{\lambda}\right)^{\beta} exp\left[-\left(\frac{t}{\lambda}\right)^{\beta}\right] m_{\tilde{t}_i}(t)dt}{\int exp\left[-\left(\frac{t}{\lambda}\right)^{\beta}\right] m_{\tilde{t}_i}(t)dt}\right] \quad (15)$$

By equalizing the derivative to zero, we obtained formulas that cannot be solved by the usual methods. Hence, we used the Newton-Ravison method [16]. On this basis, the fuzzy survival function was determined according to the formula below:

$$\tilde{\hat{S}}(t) = exp\left(-\left(\frac{t}{\hat{\lambda}}\right)^{\hat{\beta}}\right) \quad (16)$$

In this method, the upper and lower limits of the maximum likelihood estimators for fuzzy survival times are found by the following algorithm [17]:

- The values for δ are taken from 0 to 1 with an increment Δ ∈ (0; 1).
- For a given value of δ all δ-cuts of the fuzzy observations are determined.
- Taking values from the δ-cuts to get hypothetical classical samples.
- From these hypothetical classical samples at a given level δ, calculate the classical estimates.
- The lowest value and highest value of the normal powers calculated in step d are the parameters of the membership function's ends at each δ -cut.

On this basis, and by the classical maximum likelihood estimators $\hat{\lambda}, \hat{\beta}$ in formulas (7) and (6) respectively, the set of δ -cut is for each estimator:

$$C_\delta\left(\tilde{\hat{\lambda}}\right) = [\underline{\lambda}_\delta, \overline{\lambda}_\delta],\ \forall\delta \in (0,1] \tag{17}$$

and

$$C_\delta\left(\tilde{\hat{\beta}}\right) = \left[\underline{\beta}_\delta, \overline{\beta}_\delta\right],\ \forall\delta \in (0,1] \tag{18}$$

the set of δ -cut of the fuzzy survival function are according to the following formula:

$$C_\delta\left[\tilde{\hat{S}}(t)\right] = \left[exp\left[-\left(\frac{t}{\underline{\lambda}_\delta}\right)^{\overline{\beta}_\delta}\right],\ exp\left[-\left(\frac{t}{\overline{\lambda}_\delta}\right)^{\underline{\beta}_\delta}\right]\right] \tag{19}$$

B. Buckley Approach

In the case of large samples and according to the central limit theorem, the estimators are normally distributed. Hence, the minimum and maximum limits for the survival function will be as shown below [18]:

$$(\hat{S}(t) - Z\alpha_{/2}\sqrt{var\left(\hat{S}(t)\right)} \quad + Z\alpha_{/2}\sqrt{var\left(\hat{S}(t)\right)} \tag{20}$$

Where $\hat{S}(t)$: the maximum likelihood estimator of the survival function. $var\left(\hat{S}(t)\right)$: the variance of the maximum likelihood estimator of the survival function, and is calculated by the formula:

$$var\left(\hat{S}(t)\right) = \frac{\sum_{i=1}^{n}\left(\hat{s}(t_i)-\bar{\hat{s}}(t_i)\right)^2}{n-1} \tag{21}$$

$Z\alpha_{/2}$: tabular value of the normal distribution at the error value of the first type. At different values for the first type error $0.01 \le \alpha \le 1$, we obtained the confidence intervals for the survival function $\left[\hat{S}_{1\alpha}(t), \hat{S}_{2\alpha}(t)\right]$. The similarity between the confidence interval for the survival function and the triangular membership function are as follows:

$$a_1 = \hat{S}(t) - Z\alpha_{/2}\sqrt{var\left(\hat{S}(t)\right)} \tag{22}$$

$$a_2 = \hat{S}(t)$$

$$a_3 = \hat{S}(t) + Z\alpha_{/2}\sqrt{var\left(\hat{S}(t)\right)} \tag{23}$$

The fuzzy interval of the survival function is:

$$C_{\hat{S}(t)}[\delta] = \left[\hat{S}_{1\delta}(t), \hat{S}_{2\delta}(t)\right], \forall\delta \in (0,1] \tag{24}$$

III. Results and Discussion

To compare the estimators of the maximum likelihood of the survival function, we applied different fuzzy methods. The simulation experiments were designed at different sample sizes. The set of default values were chosen for the parameters of the probability density function of the Weibull distribution. The fuzzy degrees were selected in data (0, 0.5, 1). The chosen survival times was t = 2,4,6,8,10,12. To determine the best fuzzy method for the maximum likelihood estimator of the fuzzy survival function is based on the standard (AMSE) for the estimators' lower and upper limits. It should be noted that the results of these experiments were obtained using the (MATLAP program -R2017a).

A. Develop Simulation Experiment

1) The first stage: the stage of choosing default values. Two hypothetical values can be chosen for each parameter $(\lambda = 25{,}50)(\beta = 0.8{,}1.8)$. Accordingly, four models were formed for the probability density function to distribute Weibull from these parameters' combinations. The samples size can be chosen $(n = 10{,}30{,}75{,}150)$ and specify a ratio. Then, we observed 25% of each sample size, repeat these trials $(q = 10000)$.

2) The second stage: the stage of generating random variables. At this stage, the random variables of the simulation model (data) were generated, according to the following formula:

$$t_i = \lambda(-ln(1 - u_i))^{\frac{1}{\beta}} \tag{25}$$

3) The third stage: the estimating stage. In this stage, the fuzzy survival function is estimated for the lower and upper limits, according to the formula below:

$$\check{\hat{S}}(t) = \frac{\sum_L^q \tilde{\hat{S}}_L(t)}{q} \tag{26}$$

We will denote the fuzzy survival function estimator according to each fuzzy method with certain symbols. $SM_D(t)$: the maximum likelihood estimation of fuzzy survival function according to the fuzzy logic definition approach formula (17). $SM_C(t)$: the maximum likelihood estimation of the fuzzy survival function according to the level δ - cut approach formula (20). $SM_{Bu}(t)$: the maximum likelihood estimation of the fuzzy survival function according to the Buckley formula (26). According to Buckley's approach, the values of the fuzzy survival function estimators according to the level δ - cut approach are equal to the values of the fuzzy survival function estimators.

4) The fourth stage: the comparison stage, is based on the AMSE values for the lower and upper limits values for the fuzzy survival function values that were calculated in the previous stage at each limit and equal to:

$$MSE\left(\tilde{\hat{S}}(t)\right) = \frac{1}{q}\sum_L^q\left(\tilde{\hat{S}}_L(t) - S(t)\right)^2 \tag{27}$$

Where $MSE\left(\tilde{\hat{S}}(t)\right)$: the mean squared error of the fuzzy survival function $\tilde{\hat{S}}(t)$. q: the number of iterations per experiment (q=1,2,…10000).

B. The Results of Simulation Experiments

In this section, simulation results are presented to compare the maximum likelihood estimators of the fuzzy survival function for the Weibull distribution in various fuzzy approaches approved. It aims to obtain the best estimator having the less average mean squared error (AMSE) at fuzzy degrees (0,0.5,1) for all models and samples size studied at the various values of the selected survival time. Table 1 below is the presentation of the results of simulation experiments.

TABLE I
AVERAGE MEAN SQUARED ERROR VALUES OF THE FUZZY SURVIVAL FUNCTION FOR ALL MODELS AND ALL SAMPLE SIZES AT ($\Delta = 0$)

		($\lambda = 25, \beta = 0.8$)first model			($\lambda = 25, \beta = 1.8$)*second model*		
N	t	SM_D	SM_C	SM_{Bu}	SM_D	SM_C	SM_{Bu}
10	2	0.018561	0.017895	0.043527	0.004091	0.003730	0.028797
	4	0.020368	0.019946	0.044455	0.010567	0.009659	0.029027
	6	0.019414	0.019366	0.045435	0.016338	0.014993	0.029759
	8	0.017561	0.017873	0.046377	0.020457	0.018887	0.031241
	10	0.015510	0.016121	0.047239	0.022920	0.021334	0.033572
	12	0.013579	0.014419	0.048007	0.024188	0.022749	0.036682
30	2	0.018841	0.019176	0.047894	0.003662	0.003630	0.025995
	4	0.020284	0.020504	0.048184	0.009501	0.009402	0.026084
	6	0.018688	0.018753	0.048472	0.014728	0.014569	0.026366
	8	0.016051	0.015994	0.048732	0.018399	0.018227	0.026940
	10	0.013173	0.013043	0.048955	0.020288	0.020181	0.027850
	12	0.010443	0.010290	0.049142	0.020539	0.020589	0.029071
75	2	0.019264	0.019319	0.048572	0.003724	0.003728	0.026617
	4	0.020480	0.020516	0.048674	0.009670	0.009671	0.026652
	6	0.018510	0.018524	0.048772	0.014973	0.014966	0.026764
	8	0.015481	0.015480	0.048862	0.018613	0.018606	0.026991
	10	0.012253	0.012245	0.048937	0.020274	0.020286	0.027351
	12	0.009239	0.009232	0.048999	0.020007	0.020069	0.027835
150	2	0.019206	0.019306	0.048619	0.003740	0.003834	0.027811
	4	0.020422	0.020490	0.048676	0.009721	0.009962	0.027829
	6	0.018449	0.018466	0.048733	0.015055	0.015410	0.027884
	8	0.015406	0.015374	0.048785	0.018694	0.019084	0.027997
	10	0.012149	0.012086	0.048829	0.020297	0.020619	0.028174
	12	0.009099	0.009022	0.048867	0.019884	0.020052	0.028411
		($\boldsymbol{\lambda = 50, \beta = 0.8}$)third model			($\boldsymbol{\lambda = 50, \beta = 1.8}$)fourth model		
N	t	SM_D	SM_C	SM_{Bu}	SM_D	SM_C	SM_{Bu}
10	2	0.013900	0.014357	0.019402	0.001179	0.001102	0.002396
	4	0.017691	0.018187	0.019769	0.003516	0.003286	0.002412
	6	0.019283	0.019708	0.020195	0.006285	0.005874	0.002466
	8	0.019823	0.020127	0.020647	0.009112	0.008519	0.002590
	10	0.019767	0.019930	0.021104	0.011783	0.011031	0.002815
	12	0.019350	0.019366	0.021555	0.014177	0.013299	0.003172
30	2	0.015099	0.015249	0.020978	0.001274	0.001227	0.002639
	4	0.018967	0.019129	0.021095	0.003811	0.003671	0.002643
	6	0.020289	0.020427	0.021229	0.006823	0.006572	0.002658
	8	0.020373	0.020471	0.021368	0.009897	0.009535	0.002691
	10	0.019755	0.019807	0.021507	0.012781	0.012319	0.002752
	12	0.018722	0.018726	0.021641	0.015322	0.014780	0.002847
75	2	0.015267	0.015194	0.020803	0.001253	0.001263	0.002741
	4	0.019142	0.019061	0.020848	0.003752	0.003781	0.002743
	6	0.020413	0.020342	0.020899	0.006721	0.006768	0.002751
	8	0.020409	0.020356	0.020952	0.009751	0.009827	0.002769
	10	0.019681	0.019648	0.021005	0.012596	0.012692	0.002803
	12	0.018523	0.018510	0.021057	0.015101	0.015213	0.002854
150	2	0.015416	0.015349	0.021101	0.001284	0.001282	0.002802
	4	0.019307	0.019228	0.021122	0.003847	0.003842	0.002803
	6	0.020549	0.020476	0.021147	0.006893	0.006884	0.002808
	8	0.020491	0.020434	0.021172	0.010003	0.009989	0.002819
	10	0.019695	0.019661	0.021196	0.012918	0.012899	0.002839
	12	0.018463	0.018451	0.021221	0.015475	0.015454	0.002872

TABLE II
AVERAGE MEAN SQUARED ERROR VALUES OF THE FUZZY SURVIVAL FUNCTION FOR ALL MODELS AND ALL SAMPLE SIZES AT ($\Delta = 0.5$)

		$(\boldsymbol{\lambda = 25, \beta = 0.8})$first model			$(\boldsymbol{\lambda = 25, \beta = 1.8})$second model		
N	t	$\boldsymbol{SM_D}$	$\boldsymbol{SM_C}$	$\boldsymbol{SM_{Bu}}$	$\boldsymbol{SM_D}$	$\boldsymbol{SM_C}$	$\boldsymbol{SM_{Bu}}$
10	2	0.004018	0.010148	0.011332	0.000353	0.001738	0.007217
	4	0.005673	0.014930	0.012260	0.001664	0.008612	0.007446
	6	0.006401	0.017109	0.013240	0.003761	0.019484	0.008179
	8	0.006706	0.017873	0.014181	0.006352	0.031723	0.009660
	10	0.006815	0.017836	0.015044	0.009235	0.042966	0.011991
	12	0.006842	0.017347	0.015812	0.012286	0.051725	0.015101
30	2	0.003771	0.011310	0.012127	0.000259	0.001777	0.006506
	4	0.004901	0.016173	0.012417	0.001190	0.008869	0.006593
	6	0.004994	0.017982	0.012705	0.002595	0.020195	0.006876
	8	0.004642	0.018197	0.012965	0.004154	0.032992	0.007451
	10	0.004124	0.017563	0.013189	0.005614	0.044621	0.008361
	12	0.003583	0.016500	0.013375	0.006845	0.053292	0.009581
75	2	0.003812	0.011359	0.012197	0.000256	0.001741	0.006657
	4	0.004778	0.016160	0.012298	0.001161	0.008709	0.006692
	6	0.004652	0.017846	0.012397	0.002467	0.019882	0.006803
	8	0.004080	0.017916	0.012486	0.003796	0.032533	0.00703
	10	0.003369	0.017141	0.012562	0.004835	0.043991	0.007391
	12	0.002670	0.015952	0.012624	0.005420	0.052405	0.007875
150	2	0.003770	0.011412	0.012184	0.000256	0.001790	0.006954
	4	0.004704	0.016234	0.012241	0.001154	0.008978	0.006971
	6	0.004545	0.017915	0.012298	0.002437	0.020516	0.007027
	8	0.003942	0.017965	0.012350	0.003710	0.033571	0.007140
	10	0.003198	0.017164	0.012395	0.004642	0.045353	0.007317
	12	0.002469	0.015947	0.012432	0.005059	0.053922	0.007554
		$(\boldsymbol{\lambda = 50, \beta = 0.8})$third model			$(\boldsymbol{\lambda = 50, \beta = 1.8})$fourth model		
N	t	SM_D	SM_C	SM_{Bu}	SM_D	SM_C	SM_{Bu}
10	2	0.002272	0.006288	0.005012	4.98E-05	0.000283	0.000601
	4	0.003751	0.010547	0.005379	0.000265	0.001599	0.000616
	6	0.004774	0.013457	0.005805	0.000673	0.004188	0.000671
	8	0.005501	0.015445	0.006256	0.001265	0.007979	0.000794
	10	0.006028	0.016776	0.006714	0.002021	0.012746	0.001019
	12	0.006413	0.017625	0.007165	0.002915	0.018184	0.001375
30	2	0.002447	0.006947	0.005297	5.27E-05	0.000291	0.000660
	4	0.003817	0.011537	0.005415	0.000277	0.001649	0.000664
	6	0.004577	0.014562	0.005549	0.000691	0.004326	0.000679
	8	0.004957	0.016524	0.005688	0.001270	0.008255	0.000712
	10	0.005089	0.017735	0.005826	0.001972	0.013203	0.000773
	12	0.005058	0.018401	0.005960	0.002747	0.018849	0.000868
75	2	0.002455	0.007056	0.005221	4.96E-05	0.000322	0.000684
	4	0.003783	0.011721	0.005265	0.000259	0.001828	0.000687
	6	0.004476	0.014786	0.005317	0.000643	0.004802	0.000696
	8	0.004773	0.016758	0.005370	0.001172	0.009171	0.000714
	10	0.004812	0.017957	0.005423	0.001804	0.014675	0.000747
	12	0.004683	0.018595	0.005474	0.002488	0.020953	0.000799
150	2	0.002481	0.006986	0.005285	5.13E-05	0.000326	0.000701
	4	0.003804	0.011577	0.005306	0.000268	0.001854	0.000702
	6	0.004473	0.014572	0.005331	0.000663	0.004872	0.000707
	8	0.004735	0.016483	0.005355	0.001206	0.009306	0.000718
	10	0.004734	0.017627	0.005381	0.001847	0.014890	0.000739
	12	0.004564	0.018217	0.005404	0.002531	0.021257	0.000771

TABLE III

AVERAGE MEAN SQUARED ERROR VALUES OF THE FUZZY SURVIVAL FUNCTION FOR ALL MODELS AND ALL SAMPLE SIZES AT ($\Delta = 1$)

n	t	SM_D	SM_C=SM_{Bu}	SM_D	SM_C=SM_{Bu}	SM_D	SM_C=SM_{Bu}	$\boldsymbol{SM_D}$	SM_C=SM_{Bu}
10	**2**	0.000581	0.000600	2.69E-05	2.26E-05	0.000211	0.000215	1.53E-06	1.45E-06
	4	0.001446	0.001528	0.000299	0.000252	0.000580	0.000581	1.82E-05	1.71E-05
	6	0.002334	0.002509	0.001157	0.000985	0.001017	0.001008	7.67E-05	7.16E-05
	8	0.003165	0.003449	0.002847	0.002467	0.001484	0.001459	0.000209	0.000195
	10	0.003908	0.004312	0.005423	0.004797	0.001963	0.001917	0.000452	0.000420
	12	0.004552	0.005081	0.008731	0.007907	0.002439	0.002368	0.000836	0.000776
30	**2**	0.000167	0.000204	7.42E-06	8.68E-06	6.72E-05	7.05E-05	5.96E-07	3.94E-07
	4	0.000414	0.000495	8.44E-05	9.71E-05	0.000183	0.000188	7.13E-06	4.63E-06
	6	0.000663	0.000783	0.000334	0.000379	0.000318	0.000322	3E-05	1.93E-05
	8	0.000893	0.001043	0.000849	0.000954	0.000460	0.000461	8.22E-05	5.26E-05
	10	0.001094	0.001266	0.001678	0.001864	0.000604	0.000599	0.000177	0.000113
	12	0.001264	0.001453	0.002808	0.003084	0.000745	0.000733	0.000327	0.000208
75	**2**	6.14E-05	7.15E-05	2.77E-06	3.42E-06	2.65E-05	2.66E-05	2.05E-07	2.14E-07
	4	0.000151	0.000173	3.13E-05	3.82E-05	7.18E-05	7.11E-05	2.44E-06	2.53E-06
	6	0.000242	0.000272	0.000124	0.000149	0.000125	0.000122	1.02E-05	1.06E-05
	8	0.000325	0.000361	0.000313	0.000377	0.000180	0.000175	2.8E-05	2.88E-05
	10	0.000397	0.000436	0.000616	0.000737	0.000236	0.000228	6.02E-05	6.18E-05
	12	0.000457	0.000499	0.001026	0.001222	0.000290	0.000280	0.000111	0.000114
150	**2**	3.22E-05	3.91E-05	1.48E-06	1.72E-06	1.26E-05	1.28E-05	9.36E-08	1.34E-07
	4	8E-05	9.61E-05	1.7E-05	1.92E-05	3.43E-05	3.4E-05	1.11E-06	1.57E-06
	6	0.000128	0.000153	6.74E-05	7.47E-05	5.95E-05	5.83E-05	4.69E-06	6.56E-06
	8	0.000172	0.000205	0.000172	0.000187	8.61E-05	8.35E-05	1.28E-05	1.78E-05
	10	0.000211	0.000249	0.000339	0.000365	0.000113	0.000109	2.76E-05	3.82E-05
	12	0.000243	0.000287	0.000565	0.000601	0.000139	0.000133	5.1E-05	7.02E-05

TABLE IV

FREQUENCIES AND PERCENTAGE FOR THE PREFERENCE OF EACH FUZZY METHODS FOR ESTIMATING THE FUZZY SURVIVAL FUNCTION AT THE VARIOUS SAMPLE SIZES

n	SM_D		SM_C		SM_{Bu}	
	N	%	Nr	%	N	%
10	37	%51.39	18	%25	17	%23.61
30	46	%63.89	13	%18.06	13	%18.06
75	48	%66.67	14	%19.44	10	%13.89
150	49	%68.06	12	%16.67	11	%15.28
Total	180	%62.50	57	%19.79	51	%14.69

The comparison was made between the fuzzy approaches used in our research (definition of fuzzy logic, level δ- cut, Buckley). It was found that the fuzzy logic definition method of the maximum likelihood method had priority in all the sample sizes examined compared to the two (level δ- cut, Buckley). The same values were given for the average mean square error (AMSE) when (1 = δ). Table 4 shows the advantage of the maximum likelihood approach for estimating the fuzzy survival function at different sample sizes.

C. Collection of Study Data

It is known that estimating the survival functions of any disease, in general, requires providing unique survival data for patients with that disease. The data represented survival interval for each patient's from the date of the illness (the date of the first review) to the event of death or recovery. Survival times for (108) patients were obtained for kidney failure patients; among sixteen patients died during censoring from educational hospitals (Baghdad, Al-Kindi, Al-Karama, Al-Kadhimiya). The reality of recording the survival times indicates that there is uncertainty in the survival times. This means that the survival times are fuzzy numbers that have membership functions. The same values of the fuzzy factor $\tilde{z} = ([0, \infty), m_{\tilde{z}})$ were adopted. In the empirical aspect, it represents an actual triangular number whose mean value is $(z = 1)$, and the values of the fuzzy factor have been assumed $(Z_{min} = 0.5)(Z_{max} = 1.5)$

D. Goodness of Fit Test for the Data Distribution

To knowing the distribution of survival times for patients with renal failure that we obtained, a survival time data test was performed using the statistical application (Easy Fit 5.6 Professional), for the following test hypothesis:

- H_0: lifetimes are Weibull distributed.
- H_1: lifetimes are not Weibull distributed.

As the values of the calculated statistics for the survival times in all the tests were smaller than the critical value for that test at the various values of the level of significance α. Therefore, the null hypothesis states that the survival times data follows the Weibull distribution, and the following table showing the test result is accepted:

TABLE V
VALUES OF GOODNESS OF FIT TEST FOR FUZZY SURVIVAL DATA

Weibull					
Kolmogorov-Smirnov					
Sample Size	108				
Statistic	0.09032				
P-Value	0.32206				
Rank	7				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	0.10325	0.11768	0.13067	0.14607	0.15675
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size	108				
Statistic	0.98552				
Rank	3				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No
Chi-Squared					
Degree of freedom	6				
Statistic	4.5919				
P-Value	0.59712				
Rank	1				
□	0.2	0.1	0.05	0.02	0.01
Critical Value	8.5581	10.645	12.592	15.033	16.812
Reject?	No	No	No	No	No

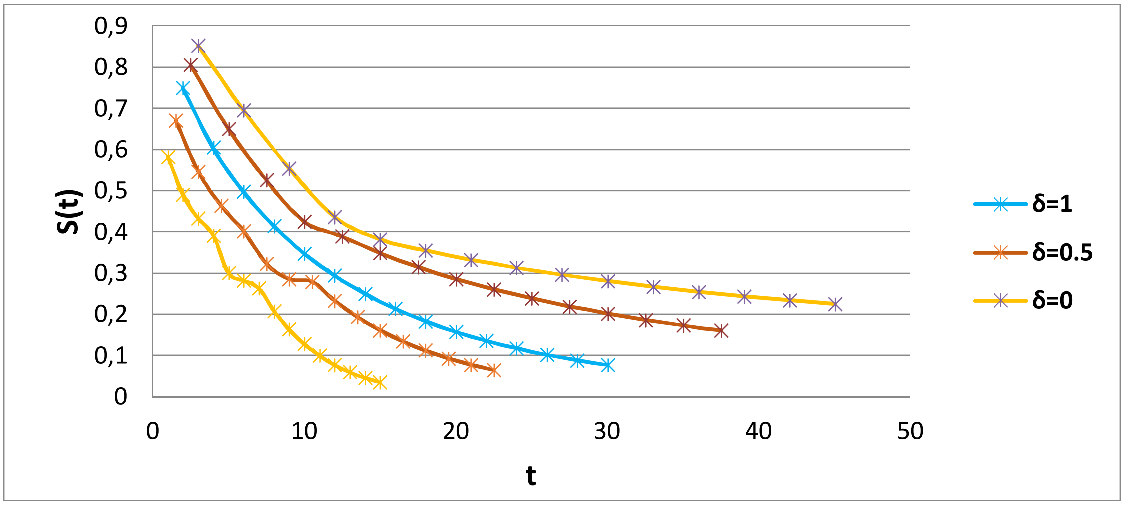
E. *Estimators for the fuzzy survival function*

We applied the maximum likelihood estimator formula using the fuzzy logic definition approach defined in formula (17) to renal failure patients' survival times and by using the program (MATLAB-R2017a). The upper and lower survival limits have been calculated at different survival times from (2-30) days. Results obtained are shown in Table (6) below:

TABLE VI
LOWER AND UPPER BOUNDS ESTIMATORS FOR FUZZY SURVIVAL PROBABILITY AT DIFFERENT FUZZY DEGREES FOR SURVIVAL TIMES (2-30) DAYS

$\tilde{t}_i$	$\hat{\tilde{S}}M_D$		$\hat{\tilde{S}}M_D$		$\hat{\tilde{S}}M_D$
	δ = 0		δ = 0.5		δ = 1
	Lower	Upper	Lower	Upper	Lower=upper
2	0.581539	0.852751	0.670918	0.805455	0.750474
4	0.489866	0.695296	0.547254	0.650378	0.604876
6	0.432513	0.555003	0.464255	0.525648	0.497699
8	0.390840	0.436422	0.402293	0.425094	0.414586
10	0.300374	0.382072	0.323315	0.389161	0.348349
12	0.329767	0.355154	0.286101	0.348797	0.294629
14	0.262985	0.332445	0.280058	0.314835	0.250515
16	0.208323	0.312884	0.233339	0.285799	0.213944
18	0.164029	0.295769	0.194397	0.260667	0.183396
20	0.128445	0.280607	0.161943	0.238695	0.157723
22	0.100073	0.267039	0.134899	0.219328	0.136032
24	0.077604	0.254796	0.112364	0.202141	0.117625
26	0.059918	0.243671	0.093588	0.186797	0.101944
28	0.046073	0.233501	0.077947	0.173030	0.088538
30	0.035291	0.224156	0.064916	0.160622	0.077043

It is clear From Table 6 that for each survival time, the lower limits of survival probabilities increase while the upper limits of that survival probabilities decrease with increasing degree of fuzzy when fuzzy degree (δ = 0,0.5). Besides, the lower and the upper survival probability are equal when (δ = 1), and it falls within the lower and upper limits of the survival probabilities when fuzzy degree (δ = 0,0.5). This is consistent with the behavior of the trigonometric membership function used in our research. The behavior of the lower and the upper survival probability limits at all degrees of uncertainty (fuzzy). The data was consistent with the behavior survival theory by decreasing the survival probabilities to increase the survival times, as shown in Figure (1).



$(\delta = 0,0.5,1)$

Fig.1 lower and the upper limits for survival probability at survival degrees

IV. Conclusion

This study investigated and searched the literature on fuzzy methods and their application in various survival theory fields. The investigation included estimating the fuzzy survival function in any known estimation methods (parametric or non-parametric). This study also covered a maximum likelihood method used. This study found that none of the Fuzzy methods have been studied and applied (Definition Fuzzy logic, cut level-δ, Buckley). By estimating the survival function in Iraq and any other Arab countries, we performed an estimate of the fuzzy survival probability. It aims to severe diseases (kidney failure), especially with the significant progress in statistical software applications.

We concluded from the experimental aspect the superiority of defining fuzzy logic in finding a maximum likelihood estimator for the fuzzy survival function over other fuzzy methods. Based on the experimental aspect results, we applied the formula of fuzzy survival function according to defining a fuzzy logic method on survival times data for patients with renal failure at different fuzzy degrees. It was found that the different fuzzy degrees (δ) in data affect the maximum likelihood estimator of the survival function. Hence, the probability of survival at each survival time is changed by changing the data's degrees of uncertainty (δ). Besides, values of shape and measurement parameters for Wiebull distribution affect the survival function's lower and upper limits. The survival probability was proportional to the shape and measurement parameter values.

The applied aspect showed that minimum and maximum survival probabilities are equal for patients with renal failure using the maximum likelihood estimator. Definition fuzzy logic method showed the case of complete fuzzy data (δ = 1). The study found out that the survival probability for (t = 2) days was equal to 0.750474. This probability decreased until reached the survival probability for (t=30) becomes equal to (0.077043). Finally, we expect that this study will be the basis for extended future studies due to the disease's seriousness, and the probability that the patient alive for one month does not exceed 8%. Therefore, we recommend providing modern dialysis machines (portable dialysis devices) to reduce patients' suffering from kidney failure.

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