

## Employing Several Methods to Estimate the Generalized Liu Parameter in Multiple Linear Regression Model

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**Abstract**— Multiple linear interferences are a fundamental obstacle in many standard models. This problem appears as a result of linear relationships between two explanatory variables or more. Simulation results show that the generalized Liu regression model was the best and that the contraction parameter proposed was more efficient than the methods presented. As the error variance increases, the value (MSE) increases. When this problem exists in the data, the estimator of the ordinary least squares method will fail because one of the basic assumptions of the method has not been fulfilled. The normal least squares, which state that there is no linear correlation between the explanatory variables, will not get an estimator with the Best Linear Unbiased Estimator (BLUE) feature. The least-squares regression method and the generalized Liu regression method were compared by taking several methods for the generalized Liu parameters and selecting the best contraction parameter for the Liu regression model. The study aims to address the problem of multiple linear interferences by using the general Liu estimator and making a comparison between the methods for estimating the Liu parameter, where several methods were presented, and the best method for estimating the Liu parameter was chosen according to the standard of the sum of error squares as well as a comparison between these methods and the conventional method. Simulation results showed that the generalized Liu coefficient estimate was the best for having the lowest values (MSE) and that the best shrinkage parameter is (G4), the work-based approach.

**Keywords**— Unbiased estimator; generalized Liu; regression; shrinkage parameter.

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### I. INTRODUCTION

The regression analysis has become one of the most widely used statistical tools for multi-factor data analysis. It is desirable because it provides an easy and understandable method for investigating the semantic relationships between variables. The standard method in regression analysis is to use a sample of data to estimate the proposed relationship using statistics such as T, F, and R<sup>2</sup>. We applied regression analysis as a set of data analysis methods to help understand the internal relationships between a given set of variables. Multiple linear regression is the relationship between the interpreted variables and the dependent variable, and when the data is a normal distribution of the interpreted variables and the dependent variable [1], [2]. In linear regression, the ordinary least squares (OLS) estimator is used to estimate the unknown regression coefficients (LRM). The explanatory factors are believed to be unrelated to the LRM [3].

Nevertheless, a regular linear connection may discover that variables that lead to multicollinearity are the ones that must be explained; it is tricky [4].

The estimation theory is of great importance in practical applications. The main objective of any estimation process is to reach the best estimate of the unknown parameter among all possible estimations. Hence, the optimal method or the best formula for estimating the unknown parameter must be chosen. The estimation of the parameters of any regression model is an interpretation, and the relationship between the response variable and several explanatory variables is in a mathematical formula. There are several different methods for estimating the parameters of the general linear regression model. Autocorrelation or the problem of multicollinearity, as the estimation process differs from one case to another depending on the presence or absence of those problems that the model suffers [5].

Many studies reported that the first to employ the term multicollinearity and give it a definition. If multicollinearity is present, the variance of the OLS estimates will be considerable, with an increased chance of erroneous wrong-sign conclusions. The estimated confidence interval's regression coefficients are larger [6]. The risk of committing a type-II mistake has increased. Also, when multiple collinearities are present, OLS estimates from several LRMs cannot be trusted. The ML estimator has numerous sources of instability. Where a linear combination of the regressors fully predicts the dependent variable, a person may have an issue of separation. Many people talk about this issue, and it results in the elimination of the ML estimator. When the ML estimations are nearly flawless, the study results show that ML estimations might be unstable.

The primary focus of this work is on a different source of instability: collinear regressors [6]. The matrix product  $XW$  is unconstrained and causes instability in the ML estimator, leading to a large variance. It is a widely used technique for dealing with multicollinearity. It is no secret that the vast majority of research for the linear model has been done and the well-known ridge regression estimator [7].

The history of polylinearity can be traced back at least to the research presented by (Frisch) in 1934 AD. The principle indicates the existence of a linear relationship between two or more explanatory variables. The method that deal with this problem is the latter regression method. It was first introduced by Hoerl and Kennard in 1970 AD. The main interest at that stage focused on finding the value of the latter regression parameter that is symbolized by  $K$ . As the use of this method leads to a reduction of MSE, the variance limit of the estimator is greater than the increase in bias square. A new logit ridge regression parameter set was proposed. In this case, the estimated parameters are complex non-linear functions of the ridge parameter  $k$ , which range from zero to infinity [8]. Another estimator is another estimate with parameters that are linear functions of the shrinkage parameter  $d$ . The usage of the Liu estimator can be credited to the fact that it offers an advantage over ridge regression [9]. The general linear regression model is:

$$z = u\phi + v \tag{1}$$

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ n \end{bmatrix}, u = \begin{bmatrix} 1 & u_{11} & u_{12} & \dots & u_{1p} \\ 1 & u_{21} & u_{22} & \dots & u_{2p} \\ 1 & u_{31} & u_{32} & \dots & u_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & u_{n1} & u_{n2} & \dots & u_{np} \end{bmatrix}, \underline{\phi} = \begin{bmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{bmatrix}, \underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_p \end{bmatrix}$$

Whereas:  $u$  with a dimensional array ( $n \times p$ ),  $\phi$  is a dimensional vector ( $p \times 1$ ) representing the unknown regression coefficients,  $v$  is a vector with a dimension ( $n \times 1$ ) representing a random error where  $E(v) = 0$ ,  $E(vv') = \sigma^2 \Gamma$  and  $\Gamma$  represent the unit array ( $n \times n$ ). The parameters  $\phi$  are found using the least-squares (OLS) method according to the following formula [10], [11]:

$$\hat{\phi}_{OLS} = (u'u)^{-1}u'z \tag{2}$$

The problem of multilinearity may not constitute a worrisome case, as the goal of building the model is to predict the values of the dependent variable based on the values of the explanatory variables because the predictive values still have a high degree of accuracy, and the values of the coefficient of determination or the modified coefficient of determination measure well to what extent the model predicts the values of The dependent variable.

However, suppose the goal of designing the multiple linear regression model is to find estimates for the parameters of the multiple linear regression model or to know the relative importance of the contribution of any of the explanatory variables to the variance of the dependent variable. In that case, the linear multiplicity is a serious problem facing the linear regression model, as it leads to the instability of the parameters of the regression model capabilities. Linearity and amplitude cause a sampling error for the estimators of the ordinary least squares method. Practically in regression analysis, researchers often encounter multicollinearity, where the problem of multicollinearity occurs when the explanatory variables are linearly related to each other.

Moreover, this problem appears in the case of the tendency of the variables to move together with increase or decrease or in the case of using time-shifting variables (Lagged Variables). When there is a problem of multiple linear relationships, then applying the least-squares method leads to a problem of inflation in the variations of the estimated regression coefficients, and diagonal elements represent this inflation. For the ( $u'u$ ), we use biased methods to eliminate this problem. There are two types of multiple linear relationships. First, the Perfect Multicollinearity here, the matrix of information is an incomplete rank, and the method of ordinary least squares cannot be applied, meaning that the regression coefficients cannot be found or determined. Second, the Semi-Perfect Multicollinearity occurs when the explanatory variables function in the same combination of other variables. Here, the information matrix ( $u'u$ ) parameter is small or close to zero regression coefficients can be found or estimated.

However, these estimates will be inaccurate. The reality of the problem being studied is not represented since the variations of the capabilities are very large [12]. There are several ways to detect linear plurality, including the Correlation Matrix, where Compute the correlation coefficients between any two explanatory variables. A high significant value of the correlation between two variables may indicate that the variables are collinear [13]. This method is easy, but it cannot produce a clear estimate of the rate of multicollinearity. Condition Number, where the correlation matrix's eigenvalues can also be used to measure the presence of multicollinearity. If multicollinearity is present in the predictor variables, one or more eigenvalues will be small. Let  $\delta_1, \delta_2, \dots, \delta_p$  be the eigenvalues of the correlation matrix. The condition number of the correlation matrix is  $cn = \delta_{max} / \delta_{min}$ , If the condition number is less than 100, there is no serious problem with multicollinearity, and if a condition number is between 100 and 1000 implies a moderate to strong multicollinearity.

Also, if the condition number exceeds 1000, severe multicollinearity is indicated. In 1967, the two researchers

### III. RESULTS AND DISCUSSION

presented Ferrar and Glaube the variance inflation factor method, which is considered one of the basic and widely used methods for detecting the problem of multilinearity. It measures the extent to which the variances of the estimated regression parameters are inflated in the presence of a linear correlation between the explanatory variables. The diagonal elements of the inverse of the system information matrix are useful in revealing Polylinearity; the variance inflation factor can be found is  $VIF=(1-H_i)^{-1}$ , Where H is the coefficient for determining the regression model of the explanatory variable  $i$  on the remaining explanatory variables.

Moreover, that its value is greater than or equal to one. The largest value of the variance inflation factor is often used as an indicator of unwanted polylinearity, and if its value exceeds 10, it is considered an indication of the possibility of an unacceptable effect of high polylinearity on the estimations of ordinary least squares. If there is a complete correlation between the independent variables, then the variance inflation factor goes to infinity. If one of the independent variables is perpendicular to the other independent variables, then the value of the inflation factor is equal to one [14], [15], [16]. Several studies were conducted on the general linear model to overcome this problem, where several methods were proposed to solve this problem. In 1993, Liu proposed a new estimator to overcome the problem of linear polymorphism, defined according to the following formula [17]:

$$\hat{\phi}_{LR} = (u'u + \Gamma)^{-1}(u'u + g\Gamma)\hat{\phi} \quad (3)$$

Whereas  $0 < g < 1$  it is a known constant parameter representing the bias parameter of the Liu estimator.  $\hat{\phi}$  represent the ordinary least squares estimator.

In 1995 Akdeniz and Kaciranlar proposed a new estimator (GL). It is defined as follows [19], [20].

$$\begin{aligned} \hat{\theta}_{GL} &= (\Lambda + \Gamma)^{-1}(u'^*z + G\hat{\theta}_{OLS})\hat{\theta}_{GL} \\ &= (\Lambda + \Gamma)^{-1}(\Lambda\hat{\theta}_{OLS} \\ &\quad + G\hat{\theta}_{OLS})\hat{\theta}_{GL} \quad (4) \\ &= (\Lambda + \Gamma)^{-1}(\Lambda + G)\hat{\theta}_{OLS}\hat{\theta}_{GL} \\ &= (\Gamma - (\Lambda + \Gamma)^{-1}(\Gamma - G))\hat{\theta}_{OLS} \end{aligned}$$

Whereas:  $G = \text{diag}(g_i)$ ,  $zero < g_i < one$ , where  $\Lambda = u'^*u^*$ ,  $u^* = uV$ ,  $V$  represents an orthogonal matrix whose columns are eigenvectors corresponding to the Eigen roots of the information matrix  $(u'u)$  and that the least squares of  $(\theta)$  are given as follows:

$$\hat{\theta}_{OLS} = (u'^*u^*)^{-1}u'^*z \quad (5)$$

Moreover, the expected value, the amount of bias, the variance matrix, and the mean square error matrix of an estimator are shown in the following equations [19], [21]:

$$\begin{aligned} E\hat{\theta}_{GL} &= (\Gamma - (\Lambda + \Gamma)^{-1}(\Gamma - G))E\hat{\theta}_{OLS} E\hat{\theta}_{GL} \\ &= (\Gamma - (\Lambda + \Gamma)^{-1}(\Gamma - G))\theta \quad (6) \end{aligned}$$

$$\text{Bias}(\hat{\theta}_{GL}) = E(\hat{\theta}_{GL} - \theta) \quad \text{Bias}(\hat{\theta}_{GL}) = (-\Lambda + \Gamma)^{-1}(\Gamma - G) \quad (7)$$

$$\begin{aligned} \text{var}(\hat{\theta}_{GL}) &= (\Gamma - (\Lambda + \Gamma)^{-1}(\Gamma - G))\text{var}(\hat{\theta}_{OLS})\Gamma \\ &\quad - (\Lambda + \Gamma)^{-1}(\Gamma - G) \quad (8) \\ &= \hat{\sigma}^2(\Gamma - K)\Lambda^{-1}(\Gamma - K)' \end{aligned}$$

$$\text{MSE}(\hat{\theta}_{GL}) = \hat{\sigma}^2(\Gamma - K)\Lambda^{-1}(\Gamma - K)' + K\theta\theta'K' \quad (9)$$

Whereas  $K = (\Lambda + \Gamma)^{-1}(\Gamma - G)$

To estimate the optimum value in equation (5), there are several methods suggested. The first and second estimators based on the work) [22] are as follows:

$$G_1 = \frac{\hat{\theta}_t^2 - 1}{\frac{1}{\delta_t} + \hat{\theta}_t^2} \quad (10)$$

$$G_2 = \text{MAX}\left(\text{zero}, \frac{\hat{\theta}_{max}^2 - 1}{\frac{1}{\delta_{max}} + \hat{\theta}_{max}^2}\right) \quad (11)$$

As  $\delta$  represent the eigen roots of the matrix  $(u'u)$ ,  $\hat{\theta}_{max}^2$  and  $\delta_{max}$  represent the largest component of  $\hat{\theta}_t^2$  and  $\delta_t$  respectively. The third estimator was based on the idea of agencies [23], [24]:

$$G_3 = \frac{\delta_t(\hat{\theta}_t^2 - \hat{\sigma}^2)}{(\delta_t\hat{\theta}_t^2 + \hat{\sigma}^2)} \quad (12)$$

The fourth estimator was proposed as follows [25]:

$$G_4 = -\left(\sqrt{\frac{\hat{\sigma}^2(1 + \delta_t)^2}{\delta_t\hat{\theta}_t^2 + \hat{\sigma}^2}} - 1\right) \quad (13)$$

The fifth and sixth estimator is based on the idea of Kibria as follows [22], [24]:

$$G_5 = \text{MAX}\left(\text{zero}, \text{Median}\left(\frac{-(1 - \hat{\theta}_t^2)}{\frac{1}{\delta_t} + \hat{\theta}_t^2}\right)\right) \quad (14)$$

$$G_6 = \text{MAX}\left(\text{zero}, \frac{1}{p}\left(\sum_{t=1}^p \frac{\hat{\theta}_t^2 - 1}{\frac{1}{\delta_t} + \hat{\theta}_t^2}\right)\right) \quad (15)$$

Finally, the seven and eighth estimator was suggested [26], [27]

$$G_7 = \text{MAX}\left(\text{zero}, \text{MAX}\left(\frac{-(1 - \hat{\theta}_t^2)}{\frac{1}{\delta_t} + \hat{\theta}_t^2}\right)\right) \quad (16)$$

$$G_8 = \text{MAX}\left(\text{zero}, \text{Min}\left(\frac{-(1 - \hat{\theta}_t^2)}{\frac{1}{\delta_t} + \hat{\theta}_t^2}\right)\right) \quad (17)$$

#### A. Simulation

In this section, the previous paragraphs were applied to generate data where explanatory variables were created by using the following equation:[28]

$$u_{kt} = \sqrt{(1 - R^2)}m_{kt} + Rm_{kp} \quad (18)$$

Where  $k = 1, 2, \dots, n$  &  $t = 1, 2, \dots, P$ ,  $R$  represent the relationship between variables,  $m_{kt}$  which are standard semi-random indices and are independent. The observational dependent variable is generated from the general regression model as follows [29], [30]:

$$z_k = \varphi_0 + \sum_{t=1}^P \varphi_t u_{kt} + u_k \quad (19)$$

Where  $\sum_{t=1}^P \varphi_t = one$  Since two values are taken to represent the sample size: 50, 100, and 200. In addition, the number of explanatory variables  $p = 5$  and  $p = 8$  is taken. Moreover, because we are concerned with the effect of the multicollinearity problem where correlation scores are more important, two values of the correlation coefficient  $\rho = (0.95, 0.99)$  are taken. Besides, five values were taken for  $\sigma^2$  2.5, 5, 10, 15, 25. The generation process was repeated 1000

times by taking different values from and where the mean error square (MSE) was calculated as follows:

$$MSE(\hat{\varphi}_r) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\varphi}_r - \varphi)^T (\hat{\varphi}_r - \varphi) \quad (20)$$

Whereas:  $\hat{\varphi}_r$  The generalized Liu estimator obtained with a different shrinkage parameter is the  $G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8$  and the least squares estimator. Table (1) show MSE values obtained from the Monte Carlo simulation study.

We conclude from the results of tables (1) that the generalized Liu estimator possesses less (MSE) compared to the OLS method in the case of multicollinearity. As the correlation coefficient value increases, the MSE value increases when all probabilities of the number of explanatory variables (p) and the sample size (n) are taken. The rated performance (GL) is also better than the OLS estimators. The higher the number of explanatory variables (p), the greater the value (MSE), and this increase affects the number of estimators. However, the estimated performance (GL) is better than that of the OLS. The best performance is the  $G_4$  performance shrinkage parameter of the Liu estimator.

TABLE I  
AVERAGE MSE VALUES FOR DIFFERENT VALUES OF  $\Sigma^2, P, N$  AND  $P$ .

$G_2$	$G_1$	OLS	$\rho$	$\sigma^2$	n	p
5.5235	5.0804	5.8536	0.95	2.5	50	5
27.2271	23.9339	28.8511	0.99			
23.5888	22.8442	23.9249	0.95	5		
113.304	108.385	115.006	0.99			
96.2477	95.286	96.692	0.95	10		
458.756	452.72	459.857	0.99			
221.526	22.0494	221.754	0.95	15		
993913	987.533	996.193	0.99			
601.485	600.380	601.871	0.95	25		
2856.9	2850.1	2857.6	0.99			
2.8954	2.6859	2.9731	0.95	2.5	100	
12.5202	11.2929	13.3406	0.99			
11.4797	11.1619	11.5989	0.95	5		
52.3056	50.3647	53.3108	0.99			
43.355	42.939	43.481	0.95	10		
212.598	210.167	213.545	0.99			
101.004	100.558	101.053	0.95	15		
491.398	488.760	492.263	0.99			
276.523	276.037	276.789	0.95	25		
1331.0	1328.2	1332.0	0.99			
10.4746	9.4087	11.1494	0.95	2.5	50	8
49.9722	42.9693	52.2022	0.99			
42.5666	40.9171	43.2316	0.95	5		
212.331	202.116	214.4291	0.99			
184.556	182.475	185.111	0.95	10		
867.509	855.18	869.735	0.99			
385.134	382.931	386.001	0.95	15		
1881.7	1868.8	1884.7	0.99			
1099.8	1097.4	1100.9	0.95	25		
5368.9	5355.3	5370.8	0.99			
4.9215	4.4694	5.1418	0.95	2.5	100	
22.1534	19.4960	23.5144	0.99			
19.2050	18.5114	19.4527	0.95	5		
93.8991	89.8405	95.2853	0.99			
77.6	76.721	77.965	0.95	10		
394.616	389.623	396.159	0.99			
179.494	178.545	179.822	0.95	15		
860.32	854.985	861.925	0.99			
489.461	488.451	489.52	0.95	25		
2424.2	2418.6	2425.7	0.99			

$G_5$	$G_4$	$G_3$	$\rho$	$\sigma^2$	n	p
4.1690	1.5296	3.30446	0.95	2.5	50	5
14.1426	6.6938	14.0953	0.99			
19.4407	5.3469	11.4704	0.95	5		
85.2945	25.4855	54.571	0.99			
87.861	21.9686	46.7490	0.95	10		
413.653	103.929	221.716	0.99			
212.678	49.485	106.036	0.95	15		
940.997	212.402	461.911	0.99			
587.77	130.981	283.938	0.95	25		
2787.4	626.640	1351.1	0.99			
2.6139	0.8668	1.6545	0.95	2.5	100	
7.9215	3.1594	6.5824	0.99			
10.1844	2.8059	5.8118	0.95	5		
40.3756	11.8366	25.3749	0.99			
40.599	9.303	20.169	0.95	10		
192.949	46.642	100.63	0.99			
97.136	22.156	47.871	0.95	15		
465.956	110.605	236.158	0.99			
271.851	60.7655	130.804	0.95	25		
1299.7	291.453	629.547	0.99			
7.2352	2.6071	5.4659	0.95	2.5	50	8
24.4115	11.6037	24.7929	0.99			
33.4257	9.4948	20.4592	0.95	5		
162.861	46.8804	101.142	0.99			
168.978	42.417	90.4	0.95	10		
794.985	190.311	410.058	0.99			
369.591	82.671	179.121	0.95	15		
1799.2	408.85	884.924	0.99			
1081.2	249.617	528.586	0.95	25		
5290.0	1185.3	2543.1	0.99			
4.3071	1.3011	2.6603	0.95	2.5	100	
12.8133	5.1525	11.1053	0.99			
16.5832	4.2779	9.1771	0.95	5		
72.4800	20.5008	44.4934	0.99			
72.017	16.579	36.054	0.95	10		
362.712	86.886	187.832	0.99			
172.74	39.179	84.783	0.95	15		
825.028	183.567	400.152	0.99			
482.018	105.278	228.429	0.95	25		
2386.5	522.422	1133.8	0.99			

  

$G_8$	$G_7$	$G_6$	$\rho$	$\sigma^2$	n	p
3.9762	5.4438	3.9805	0.95	2.5	50	5
6.4268	25.6184	6.8333	0.99			
15.6473	23.4471	16.1382	0.95	5		
26.202	111.273	38.9634	0.99			
64.874	96.097	71.639	0.95	10		
148.969	456.650	253.326	0.99			
158.185	221.362	178.029	0.95	15		
411.778	991.752	642.718	0.99			
465.270	601.321	520.815	0.95	25		
1640.8	2854.7	2200.7	0.99			
2.6009	2.8843	2.6009	0.95	2.5	100	
5.9425	12.0976	5.9735	0.99			
9.5751	11.4536	9.6109	0.95	5		
22.6125	51.7148	24.6708	0.99			
35.607	43.32	36.662	0.95	10		
98.266	211.956	124.254	0.99			
83.672	100.968	87.354	0.95	15		
255.039	490.753	329.108	0.99			
237.838	276.486	251.645	0.95	25		
824.394	1330.3	1033.6	0.99			
7.0019	10.2954	7.0027	0.95	2.5	50	8
10.5387	47.1657	11.2565	0.99			
26.8282	42.3241	27.6469	0.95	5		
43.7831	209.055	73.408	0.99			

115.858	184.283	132.828	0.95	10	
239.198	864.065	477.959	0.99		
254.068	384.851	300.891	0.95	15	
670.987	1878.4	1252.0	0.99		
797.257	1099.5	940.975	0.95	25	
2661.0	5365.6	4188.8	0.99		
4.3032	4.8974	4.3032	0.95	2.5	100
9.7039	21.4204	9.7075	0.99		
15.7464	19.1594	15.7850	0.95	5	
38.6850	92.9770	43.2455	0.99		
62.309	77.543	64.018	0.95	10	
169.497	393.651	232.28	0.99		
14.779	179.438	153.355	0.95	15	
410.045	859.360	589.328	0.99		
407.049	489.404	437.229	0.95	25	
1377.4	2423.2	1913.4	0.99		

#### IV. CONCLUSION

In this paper, the least-squares and the generalized Liu regression methods were compared by taking several generalized Liu parameters and selecting the best contraction parameter for the Liu regression model. Simulation results show that the generalized Liu regression model was the best and that the contraction parameter was more efficient than the methods presented. As the increase of the error variance  $\sigma^2$ , the increase in the value (MSE), and as the sample size increases, the value of (MSE) decreases when taking different values for each correlation coefficient, the number of explanatory variables, and error variance.

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