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Inclined Magnetic Field Effects on Chaotic Convection for Moderate Prandtl Number

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***Abstract*— Using the theory of dynamical systems, this study investigated the effects of the inclined magnetic field on chaotic behaviour in a fluid layer heated from below for moderate Prandtl number. A low-dimensional, Lorenz-like model was obtained using the Galerkin truncated approximation. The fourth-order Runge-Kutta method was employed to solve the nonlinear system. The solution shows that it is possible to delay the chaotic convection depending on the Hartmann number.**

***Keywords*— Chaotic behaviour; inclined magnetic field; Lorenz equations.**

I. Introduction

Chaotic behaviour has attracted interest due to its wide application. It can be observed in many natural systems, such as the time evolution of the magnetic field of celestial bodies, molecular vibrations, the dynamics of satellite in the solar system, the weather, in ecology and in neurons.

The transition from steady convection to chaos in a porous medium for low Prandtl number, studied by Vadasz and Olek [1], is sudden and occurs by a subcritical Hopf bifurcation producing a solitary limit cycle which may be associated with a homoclinic explosion. This finding can be recovered from a truncated Galerkin expansion (Vadasz and Olek [2]) that yields a system identical to the familiar Lorenz equations (Lorenz [3], Sparrow [4]). For the corresponding convection problem in a pure fluid, a similar approach was used by Vadasz [5] to demonstrate similar results. Vadasz and Olek [6] showed that the route to chaos occurs by a period doubling sequence of bifurcations when the Prandtl number is moderate. Jawdat and Hashim [7] showed that the onset of chaotic convection in a porous medium for low prandtl number can be enhancing by a uniform internal heat generation.

The study of magnetic field effects has important applications in physics and engineering. It includes the Hall effects, the mass spectrometer, microphone and the flow and heat transfer problems.

The effects of a magnetic field on chaotic convection in porous media for low Prandtle number were investigated by Idris and Hashim [8]. They observe that the magnetic field will delay the convective motion in a saturated porous medium fluid layer. Mahmud and Hashim [9] studied the chaotic convection in a fluid layer heated from below when a constant, vertical magnetic field was applied. They showed that the chaotic convection can be suppressed or enhanced.

The aim of the present work is to study the influence of inclined magnetic field on chaotic convection in fluid layer heated from below for moderate Prandtl number extending the work of Vadasz [5]. The truncated Galerkin approximation was applied to the governing equations to deduce an autonomous system with three ordinary differential equations. This system was used to investigate the dynamic behaviour of thermal convection in the fluid layer and to elucidate the effects of inclined magnetic field on the transition to chaos.

II. Problem Formulation and Equations

Consider an infinite horizontal fluid layer subject to gravity and heated from below with influence of inclined magnetic field B with angle ϕ. A Cartesian co-ordinate system is used such that the vertical axis z is collinear with gravity, i.e. $\hat{e}_g = -\hat{e}_z$. A linear relationship between density and temperature is assumed and can be presented as $\rho = \rho_0 [1 - \beta_* (T_* - T_c)]$ where β_* represents the thermal expansion coefficient. Also, the Boussinesq approximation is applied indicating that density variations are effected only for the gravity term in the momentum equation.

Subject to these conditions, the dimension governing equations can be written as

$$\nabla \cdot V_* = 0 \quad (1)$$

$$\rho_0\left[\frac{\partial V_*}{\partial t_*} + V_* \cdot \nabla V_*\right] = -\nabla p_* + \upsilon_* \nabla^2 V_* + \rho\vec{g} + J\times B \quad (2)$$

$$\frac{\partial T}{\partial t_*} + V_* \cdot \nabla T = \alpha_* \nabla^2 T \quad (3)$$

$$\nabla \cdot J = 0; J = \sigma\left(-\nabla\varphi + V_* \times B\right) \quad (4)$$

where V_* is the velocity, T is temperature, p_* is pressure, υ_* is fluid viscosity, α_* is thermal diffusivity, J is electric current density, B is applied magnetic field, φ is electric potential, σ is electric conductivity and ρ_0 is a reference value of density.

Garandet et al. [10] suggested that the electric potential in Eq.(4) was significantly reduced to $\nabla^2\varphi = 0$ for a 2-D steady-state situation. Since $\partial\varphi/\partial n = 0$, the unique solution is $\nabla\varphi = 0$. This means that the electric field vanishes everywhere.

The following transformations will non-dimensionalize Eqs. (1)-(4):

$$V = \frac{H_*}{\alpha_*} V_*\ ,\ p = \frac{H_*^2}{\rho_0 \alpha_*^2} p_*\ ,\ t = \frac{\alpha_*}{H_*^2} t_*\ ,$$
$$T\Delta T_c = T_* - T_c\ ,\ x = \frac{x_*}{H_*}\ ,\ y = \frac{y_*}{H_*}\ ,\ z = \frac{z_*}{H_*} \quad (5)$$

where t is the time, $(T_* - T_c)$ is the temperature variations and $\Delta T_c = (T_H - T_c)$ is the characteristic temperature difference.

The fluid layer with stress-free horizontal boundaries is considered and the temperature boundary conditions are $T=1$ at $z=0$, $T=0$ at $z=1$.

The governing equations can be represented in terms of a stream function defined by $u = -\partial\psi/\partial z$ and $w = \partial\psi/\partial x$, as for convective rolls having axes parallel to the shorter dimension (i.e. y) when $v = 0$. Applying the curl ($\nabla\times$) operator on Eq.(2) yields the following system of partial differential equations from Eqs. (1)-(3):

$$\left[\frac{1}{\Pr}\left(\frac{\partial}{\partial t} - \frac{\partial\psi}{\partial z}\frac{\partial}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial z}\right) - \nabla^2\right]\left(\nabla^2\psi\right) = Ra\frac{\partial T}{\partial x}$$
$$-(Ha)^2\left[\frac{\partial^2\psi}{\partial x^2}\cos^2\phi + 2\frac{\partial^2\psi}{\partial x\partial z}\cos\phi\sin\phi + \frac{\partial^2\psi}{\partial z^2}\sin^2\phi\right] \quad (6)$$

$$\frac{\partial T}{\partial t} - \frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \quad (7)$$

where ψ, $\Pr$, Ra and Ha are the stream functions, the Prandtl number, the Rayleigh number and the Hartmann number, respectively.

In order to obtain the solution to the nonlinear coupled system of partial differential equations in Eq. 6 and Eq. 7, we represent the stream function and temperature in the form

$$\psi = A_{11}\sin(\kappa x)\sin(\pi z) \quad (8)$$

$$T = 1 - z + B_{11}\cos(\kappa x)\sin(\pi z) + B_{02}\sin(2\pi z) \quad (9)$$

This representation is equivalent to a truncated Galerkin expansion of the solution in both the x- and z-directions. Following the Vadasz work in [5] to our case, rescaling the time and the amplitudes with respect to their convective fixed points, we have the following system of ordinary differential equations

$$\dot{X} = G\Pr(Y - X) \quad (10)$$

$$\dot{Y} = (R/G)X - Y - ((R/G) - 1)XZ \quad (11)$$

$$\dot{Z} = \lambda(XY - Z) \quad (12)$$

where $G = 1 + \frac{4(Ha)^2}{9\pi^2}\left[\left(1/\sqrt{2}\right)\cos\phi - \sin\phi\right]^2$, $R = (Ra/Ra_c)$, $Ra_c = \frac{(\kappa^2+\pi^2)^3}{\kappa^2}$ and the primes denotes time derivatives $d()/d\tau$. When $Ha=0$, system (10)-(12) reduces to the Vadasz system [5] (Eqs.(11)-(13)). System (10)-(12) is equivalent to the Lorenz equations, although with different coefficients. By using the wavenumber corresponding to the convection threshold, i.e. κ_{cr}, yields $\lambda = 8/3$ and $Ra_c = 27\pi^4/4$.

III. STABILITY ANALYSIS

The nonlinear dynamics of a Lorenz-like system (10)-(12) has been analyzed and solved for $\Pr=50$, $\lambda=8/3$ and $\phi=\pi/4$. This rescaled system has three fixed points. The first fixed point is $X_1=Y_1=Z_1=0$, corresponding to the motionless solution. The second and third fixed points corresponding to the convection solution are $X_{2,3}=Y_{2,3}=\pm1, Z_{2,3}=1$. The stability of the first fixed point is controlled by the zeros of the following characteristic polynomial equation for the eigenvalues $\alpha_i (i=1,2,3)$:

$$(-\lambda - \alpha)\left[(G\Pr + \alpha)(1+\alpha) - \Pr R\right] = 0 \quad (13)$$

α_1 and α_3 are negative and α_2 provides the stability condition for the motionless solution in the form $\alpha_2 < 0 \Leftrightarrow R < G$. Therefore the critical value of is obtained as $R_{c1} = R_{cr} = G$, which corresponds to $Ra_{cr} = (27\pi^4/4)G$. This means that there is a direct proportionality between the Hartmann number Ha and the Rayleigh number Ra with fixed inclination angle ϕ. Fig. 1 shows this direct proportionality between the Hartmann number Ha and Rayleigh number Ra for fixed inclination angle $\phi=\pi/4$.

The following cubic equation for the eigenvalues, $\alpha_i (i=1,2,3)$, controlled the stability of the second and third fixed points of the rescaled system

$$\alpha^3 + (1+\lambda+G\Pr)\alpha^2 + (G\Pr\lambda + \lambda(R/G))\alpha$$
$$+2\Pr\lambda(R-G) = 0 \quad (14)$$

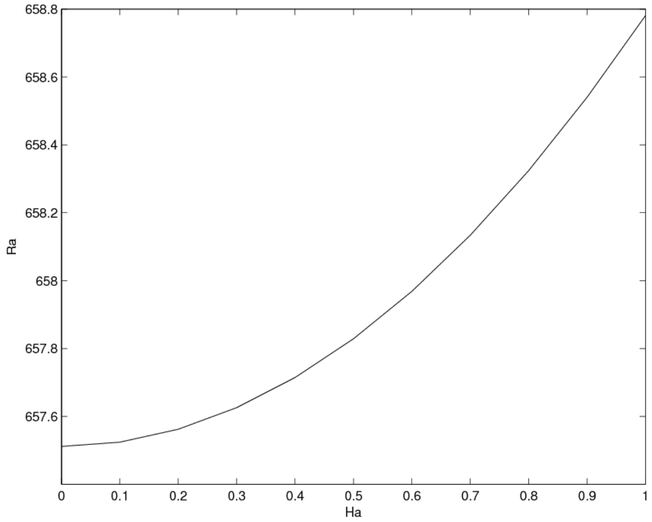


Fig. 1. A graph of Ha versus Ra showing the direct proportionality between them for fixed inclination angle $\phi=\pi/4$.

Eq.(14) yields three eigenvalues, and the smallest eigenvalue α_1 is always real and negative over the whole range of parameter values. The other two are real and negative at slightly supercritical values of R, such that the convection fixed points are stable, that is, simple nodes. These two roots move on the real axis towards the origin as the value of R increases. These roots become equal when they become a complex conjugate. In any case, they still have negative real parts, and so the convection fixed points are stable, that is, spiral nodes. Both the imaginary and real parts of these two complex conjugate eigenvalues increase and extend over the imaginary axis as the value of R increases. The real part becomes nonnegative at a value of R given by

$$R_{c2} = \frac{G^2 \Pr(3+\lambda+G\Pr)}{(G\Pr-\lambda-1)} \tag{15}$$

Relation (15) is an extension of R_0 in [5] to the inclined magnetic field case $Ha \neq 0$. At this point, the convection fixed points lose their stability and other (periodic or chaotic) solutions take over. The values of G and the values of R where the loss of stability and chaotic behaviour occurred for different values of Ha are presented in Table 1.

TABLE I
Values of G and Values of R when the Loss of Stability Occurred for Different Values of Ha

Ha	G	R (Loss of Stability)
0	1	60.07194245
0.25	1.000120722	60.08513510
0.5	1.000482888	60.12472165
0.75	1.001086498	60.19072845
1	1.001931552	60.28319935

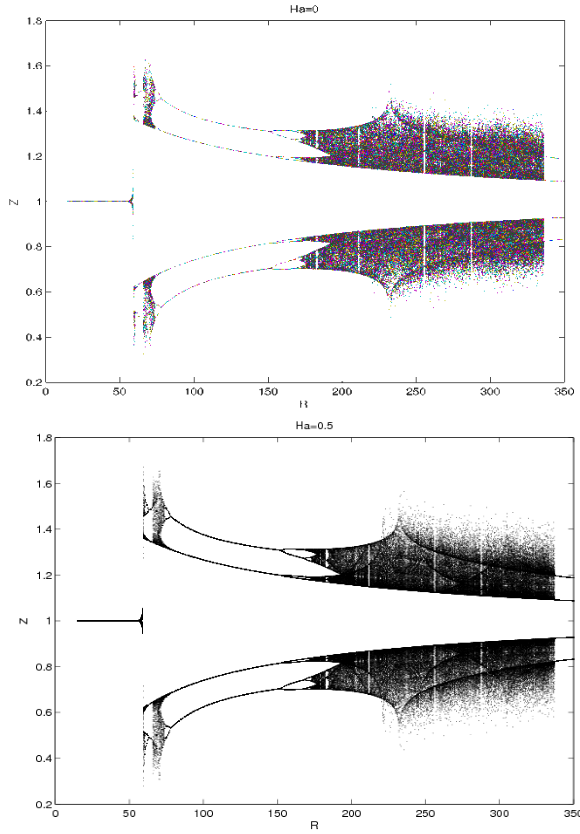
IV. Results and Discussion

In this section, some numerical simulations of the system (10)-(12) are presented for $0 \le t \le 210$. All calculations were done using MATLAB's built-in ODE45 based on the fourth-order Runge-Kutta method on double precision with step size 0.001, fixing the values $\Pr=50$, $\lambda=8/3$, $\phi=\pi/4$ and taking the initial conditions $X(0)=Y(0)=0.8$ and $Z(0)=0.92195$.

The complete solutions were computed for a wide range of R values between $0<R\le 350$ with a resolution of $\Delta R=0.5$. The computational results identified maxima and minima in the post-transient solution for each value of R, which were plotted as a function of R producing the bifurcation diagrams presented in Fig. 2 in terms of maxima and minima in the post-transient values of Z versus R.

The evolution of trajectories over time in the state space for two values of Hartmann number ($Ha=0$ and $Ha=0.5$) is presented in Fig. 3.

We can notice from Table II and Fig. 2 that the critical value of the scaled Rayleigh number R increases when Ha increases. Hence, the chaotic behaviour can be enhanced when Ha is decreasing and delayed when Ha is increasing.

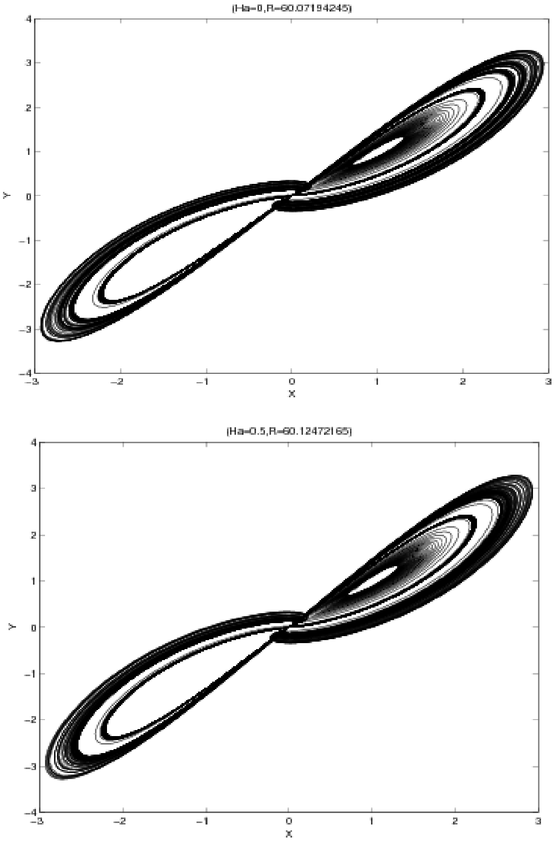


(a) (b)

Fig. 2. Bifurcation diagrams of Z versus R, representing maxima and minima of the post-transient solution of $Z(t)$ for (a) $Ha=0$ and (b) $Ha=0.5$.

V. Conclusions

In this paper, we have studied chaotic behaviour in a fluid layer subject to gravity and heated from below under the effect of an inclined magnetic field with $\phi=\pi/4$ for moderate Prandtl number. We notice that there is a direct proportionality between the Hartmann number Ha and the Rayleigh number Ra with fixed inclination angle ϕ, and the chaotic behaviour can be delayed depending on the value of Hartmann number Ha.



(a)

(b)

Fig. 3. Computational results for the evolution of trajectories over time in the state space for two values of Hartmann number (a) $Ha=0$ and (b) $Ha=0.5$.

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