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Non-dominated Sorting Harris's Hawk Multi-Objective Optimizer based on the Flush-and-Ambush Tactic

Shaymah Akram Yasear[a,1], Ku Ruhana Ku-Mahamud[a,2]

[a] School of Computing, Universiti Utara Malaysia, Sintok, Changlun, 06010, Malaysia
E-mail: [1]shayma.akram.yasear@gmail.com; [2]ruhana@uum.edu.my

***Abstract*— In this paper, a new population update strategy is proposed to overcome the limitations of the non-dominated sorting Harris's hawk multi-objective optimizer (NDSHHMO) algorithm. In the NDSHHMO algorithm, the population of hawks is updated based on the average positions of the first three best solutions in the search space. This update strategy leads to the algorithm falling into local optima due to population diversity loss, which causes poor convergence toward the true Pareto front. The proposed population update strategy is inspired by the flush-and-ambush (FA) tactic employed by the Harris's hawks in nature. The proposed algorithm is called non-dominated sorting Harris's hawks' multi-objective optimizer based on the flush-and-ambush tactic (FA-NDSHHMO). The population update strategy in the FA-NDSHHMO includes two main stages, namely, updating the position of hawks using proposed flush-and-ambush movement strategy and selecting the best hawks by using a non-dominated sorting approach to be used in the next generation. The proposed population update strategy aims to improve the search ability of the algorithm, in terms of the diversity of a non-dominated solution and convergence toward the Pareto front. To evaluate the performance of the FA-NDSHHMO algorithm, a set of 10 multi-objective optimization problems has been used. The obtained results show that the new population update strategy has improved the search ability of the FA-NDSHHMO. Furthermore, the results show superiority of the FA-NDSHHMO algorithm compared to the NDSHHMO, multi-objective grasshopper and grey wolf optimization algorithms.**

***Keywords*— swarm intelligence; metaheuristic; population-based; optimization algorithm.**

I. INTRODUCTION

Swarm intelligence-based (SI-based) metaheuristics, inspired by nature, have gained great interest in the development of new metaheuristics. The SI-based algorithms mimic natural biological evolution processes, foraging behaviours, or physical phenomena. These algorithms include, but are not restricted to, particle swarm optimization (PSO) [1], grasshopper optimization algorithm [2], grey wolf optimizer (GWO) [3] and ant colony optimization algorithm (ACO) [4]. Metaheuristics have been successfully applied to solve optimization problems in different areas such as engineering, industry and science [5], [6]. However, optimization problems in real-world applications usually include two or more conflicting objectives, where improving an objective leads to the degradation of others. The classical method for multi-objective optimization (MOO) converts the problem with multiple objectives into a single-objective optimization problem (SOP) and then uses a single objective optimization algorithm to solve it. However, this method becomes less efficient with the increasing complexity of the problem and increasing number of objectives, which drives more research towards designing more effective optimization algorithms.

SI-based optimization algorithms are inherently based on a population of multiple solutions that can generate as many solutions as possible in a single execution with the ability to find multiple solutions simultaneously. Thus, SI-based metaheuristics are useful for MOO [7]. The multi-objective SI-based metaheuristics extend single optimization algorithms to handle multi-objective optimization problems (MOPs) such as, multi-objective particle swarm optimization [8], multi-objective grasshopper optimization algorithm (MOGOA) [9] and MOGWO [10].

In Yasear and Ku-Mahamud [11] the non-dominated sorting Harris's hawk multi-objective optimizer (NDSHHMO) algorithm has been proposed. This algorithm combines the non-dominated sorting (NDS) of Deb, et al. [12] with HHMO and has shown better performance in dealing with MOPs. Updating the population of solutions is one of the main processes in the SI-based metaheuristics. In the NDSHHMO algorithm, the new position of hawks (candidate solutions) is updated based on the average positions of three leaders. These leaders represent the first, second and third best solutions in the search space, and the positions of other hawks are not considered in updating the position of hawks [13], [14].

This indicates a lack of sharing the information between the hawks in the population. In this case, the NDSHHMO algorithm will not be able to escape from local optima, due to the loss of population diversity, especially, in solving complex MOPs [13, 14]. This, in turn, leads to poor convergence toward the true Pareto front (PF) [15, 16]. Sharing of information by utilizing the experiences of all hawks during the search process is very important to accelerate and ensure the convergence and diversity of the obtained solutions [17, 18]. The cooperation between individuals during the searching process is one of the main concepts of the SI system [19]. In this context, this paper aims to improve the NDSHHO algorithm by proposing a new population update strategy that considers the diversity of the produced solutions and the contribution of all hawks in updating their position. The diversity and convergence of the FA-NDSHHMO have been evaluated through a comparison with the original NDSHHMO, MOGWO and MOGOA algorithms in solving 10 MOPs.

The organization of the paper is as follows: the NDSHHMO algorithm is presented in Section II, followed by introducing the improved NDSHHMO in Section III. The experimental design and results are provided in Sections IV and V, respectively. Concluding remarks are presented in the final section.

II. Material and Method

A. Non-dominated Sorting Harris's Hawk Multi-Objective Optimizer

The NDSHHMO algorithm [11] is an SI-based metaheuristic inspired by the social hierarchy and hunting behaviour of the Harris's hawk in nature [20]. In the NDSHHMO algorithm, each hawk can be considered as a candidate solution for a problem. Based on the number of reference points (the prey), the population is divided into groups. Each group includes three leaders (α, β and δ), and γ, the remaining hawks. The leaders are the hawks that have the three shortest distances to a reference point, Z. During the optimization process, the movement direction of the X_γ hawks is determined according to the average positions of leaders X_α, X_β and X_δ to move towards the prey. This behaviour is formulated as shown in Equation (1).

$$\vec{X}(t+1) = \vec{X}_{\mathrm{p}}(t) - \vec{A} * \vec{D} \quad (1)$$

$X = (x_1, x_2, \cdot \cdot \cdot, x_d)$ is the position vector of the hawks, while $X_{\mathrm{p}} = (x_{\mathrm{p}1}, x_{\mathrm{p}2}; \ldots, x_{\mathrm{p}d})$ is the position vector of the prey in d dimension at iteration t. $\vec{D}$ is the distance between the hawk and the prey, which is calculated for each group as shown in Equation (2). $\vec{A}$ and $\vec{C}$ are convergence factors calculated as shown in Equation (3) and (4), respectively.

$$\vec{D} = C * \vec{X}_p(t) - \vec{X}(t) \quad (2)$$

$$\vec{A} = 2\vec{a} * r_1 - \vec{a} \quad (3)$$

$$\vec{C} = 2 * r_2 \quad (4)$$

where a is the convergence parameter which decreases linearly from 2 to 0 with the number of iterations. r_1 and r_2 are random numbers in interval [0,1]. The position of hawks is updated as shown in Equations (5) and (6).

$$\vec{X}(t+1) = \frac{(\vec{X}_\alpha(t) + \vec{X}_\beta(t) + \vec{X}_\delta(t))}{3} \quad (5)$$

$$\begin{aligned} \vec{X}_\alpha(t+1) &= \vec{X}_\alpha(t) - \vec{A}_\alpha * \vec{D}_\alpha \\ \vec{X}_\beta(t+1) &= \vec{X}_\beta(t) - \vec{A}_\alpha * \vec{D}_\beta \\ \vec{X}_\delta(t+1) &= \vec{X}_\delta(t) - \vec{A}_\alpha * \vec{D}_\delta \end{aligned} \quad (6)$$

The position update strategy in NDSHHMO does not consider the positions of other hawks, X_γ, in updating the position of hawks in the search space. This leads to a loss of population diversity due to high selection pressure in which the algorithm depends only on the three best solutions to guide the search process [21]. The high selection pressure and loss of population diversity leads to poor convergence toward the true PF [22].

B. Improved Non-dominated Sorting Harris's Hawk Multi-Objective Optimizer

In any optimization algorithm, the population update strategy can be considered as the core of the algorithm, which is used to produce new solutions. This section introduces the proposed population update strategy. This strategy is integrated with the NDSHHMO algorithm to further improve the diversity of obtained solutions and the convergence toward the true PF.

The proposed population update strategy of hawks consists of two main stages. The first stage is to calculate the new position of hawks by using a new movement strategy and the second stage is to select the hawks to be used in the next generation. Fig. 1 shows the proposed population update strategy with the NDSHHMO algorithm.

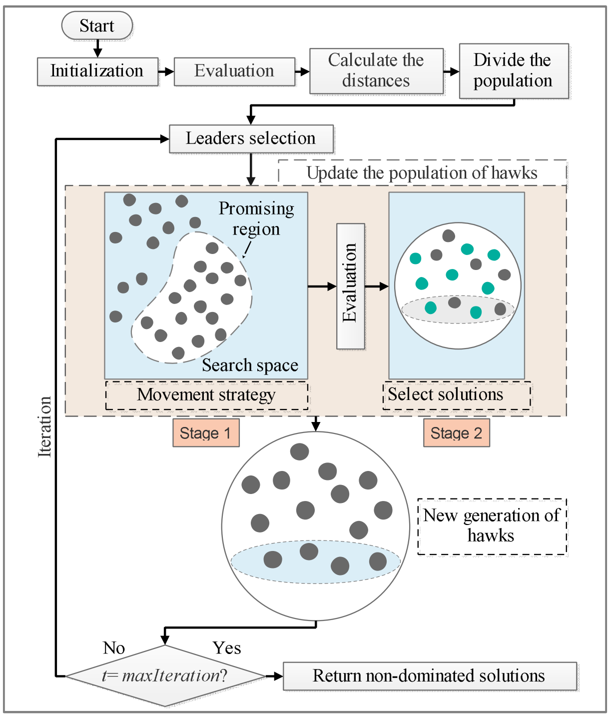

Fig. 1 Proposed population update strategy

In the first stage, a new movement strategy is used to calculate the new positions of the hawks. This movement strategy is developed based on the hunting behaviour employed by the Harris's hawks in nature. Harris's hawk attacks are quite coordinated. According to Bednarz [20], who observed Harris's hawks over a period of years, their hunting behaviour involves different tactics. These tactics vary in an unpredictable sequence, based on the changing circumstances that occur during pursuit of prey. One of these tactics is called "flush-and-ambush" [20]. This tactic is employed when a prey finds temporary refuge or cover, as illustrated in Fig. 2 [23].

Fig. 2 Flush-and-ambush tactic: The prey finds temporary refuge or cover

In the flush-and-ambush tactic (FA), the hawks are alert in watching the location where the prey disappeared. Meanwhile, one or possibly two hawks attempt to penetrate the cover. Then, when the prey is flushed, one or more of the hawks pounce and kill the prey [20]. Based on the hunting tactics of these hawks, the proposed movement strategy is formulated as shown in Equation (7).

$$\vec{X}(t+1) = \begin{cases} \dfrac{\left(\vec{X}_1(t)+\vec{X}_2(t)+\vec{X}_3(t)\right)}{3} & ;\quad \text{if } p \geq 0.5 \\ \vec{X}_{\mathrm{FA}} & ;\quad \text{otherwise} \end{cases} \tag{7}$$

where p is a random value in interval [0,1]. In the proposed movement strategy, the new position, $\vec{X}(t+1)$ is calculated based on the random-proportional rule. This rule is an action choice rule typically used in Q-learning [24]. With this rule, the action is chosen randomly with a probability of 50%. This means that the old and proposed position update strategy have exactly the same probability to be chosen to calculate the new positions of hawks. The random-proportional rule has also been used in other algorithms [25, 26]. Fig. 3 illustrates the proposed movement strategy.

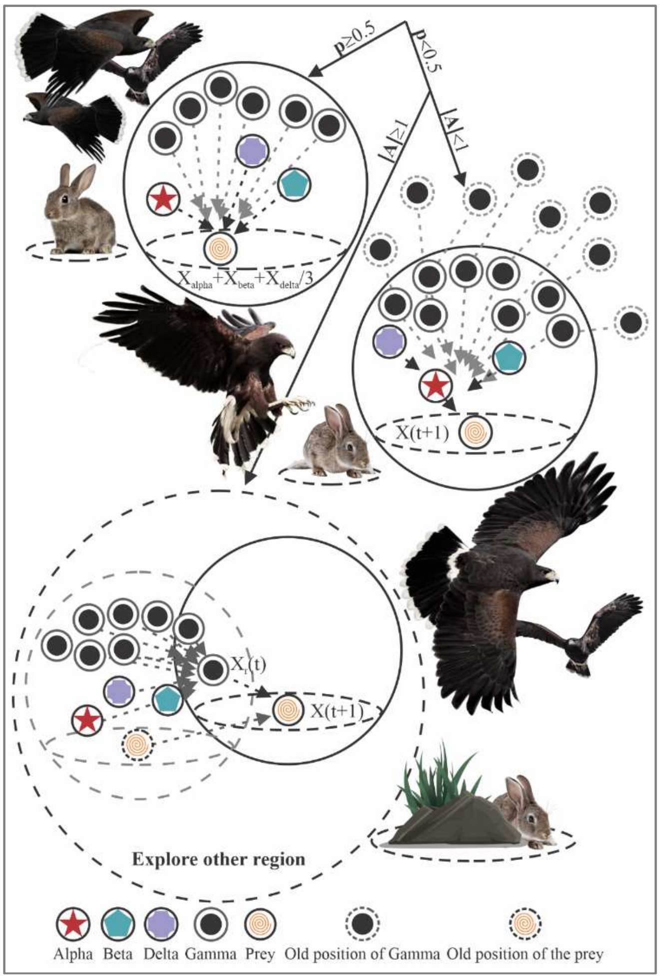

Fig. 3 Proposed flush-and-ambush movement strategy

If $p \geq 0.5$, this indicates that X_α, X_β and X_δ hawks have spotted the location of the prey. In this case, Equation (5) in the original update strategy will be employed to generate new positions of hawks according to the positions of leaders. Otherwise, if the pray escapes, the positions of hawks will be updated based on the FA movement strategy, which is represented by $\vec{X}_{\mathrm{FA}}$ value, as shown in Equation (7).

$$\vec{X}_{\mathrm{FA}}(t+1) = \begin{cases} \vec{X}_r(t) - \vec{A} * \vec{D}_r & ;\quad \text{if } |A| \geq 1 \\ \vec{X}_\alpha(t) - \vec{A} * \vec{D}_\alpha & ;\quad \text{otherwise} \end{cases} \tag{8}$$

the value of $\vec{X}_{\mathrm{FA}}$ is proportional to the absolute value of A. In this approach, the hawks move forward and backward from the prey based on the value of $|A|$. If $|A| \geq 1$, the hawks will explore the desert site looking for potential prey. If $|A| < 1$ forces the hawks to move towards the prey. In the proposed FA movement strategy, Equation (8), if $|A| \geq 1$, this indicates the prey has successfully escaped from the hawks and found temporary cover (refer Fig. 3). In this case, $\vec{X}_{\mathrm{FA}}$ is calculated using a random hawk, $\vec{X}_r$, which is selected from the current population, represented by X_γ hawks, to guide the search process. The random position of the hawk represents the exploration of different regions to find the location of the covered prey. If $|A| < 1$, this represents penetrating the cover of the prey. In this case, the X_α hawk makes the final move to kill the prey. In other words, the new position of hawks is calculated according to the $\vec{X}_{\alpha}$ hawk in a group, which represents the nearest hawk to the prey.

The second stage of updating the population of hawks requires selecting the non-dominated solution to be used in the next generation. To select the non-dominated solutions, the NDS is used [27]. In this approach, the population of parent and offspring are combined to produce a population of size $2N$. This population is sorted and classified according to the Pareto dominance relation between the solutions, forming several front levels. The individuals that have the best quality in the population are considered as a first level of frontier, F_1 and assigned the rank 1. Subsequently, these individuals are temporarily eliminated from the competition. The non-dominated individuals in the remaining population are selected to construct the second level of frontier, F_2 and assigned the rank 2. These processes are repeated until there is no individual left. In this way, the population is divided into multiple non-dominated frontiers, each defining a specific quality level. Fig. 4 illustrates the principle of NDS [12].

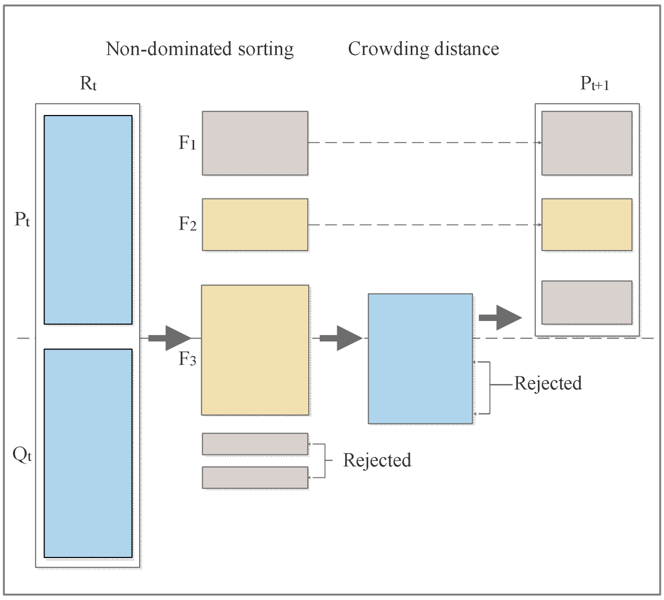

Fig. 4 Concept of the NDS approach approach

To perform NDS, the two population parents, P_t and off-spring, Q_t are combined into a single R_t population composed of $2N$ solutions. To select the best solutions, the solutions of the R_t population are sorted based on the Pareto-dominance relation between the solutions, forming several front levels namely F_1, F_2 and F_3. The solutions in the first level, F_1 are not dominated by any other solutions in the R_t population, and assigned the rank 1. The second level consists of non-dominated solutions in P-F_1 and assigned the rank 2. The third level includes P-F_1-F_2 and assigned the rank 3, and so on until all solutions are sorted into several fronts.

In general, with NDS-based algorithms, if the number of solutions in F_1 is less than the predefined population size, N, the rest will be selected from the next front, F_2. If the total number of selected solutions exceed N, the solutions of F_1 will be moved to the next generation and the rest will be selected from F_2 based on another quality criterion.

Several studies have proved the effectiveness of the NDS approach [12] with many MOO algorithms [12, 28, 29]. The NDS approach helps in improving the convergence of the algorithm towards the true PF, especially for dealing with complex MOPs with a large number of local PFs [30].

In the NDS approach, the crowding distance [12] determines which individuals will survive for the next generation. The crowding distance estimates the degree of a solution crowding by calculating the average distance of its two neighbouring solutions. Solutions that are on the edge of the PF have only one neighbour, but they are the most diverse of the border, so they obtain high values and, consequently, are at the top of the order. The solutions with bigger crowding distance are preferred. However, in some cases, the crowding distance approach cannot be used to select appropriate solutions, which may affect the diversity of solutions. Fig. 5 illustrates the limitation of the crowding distance approach.

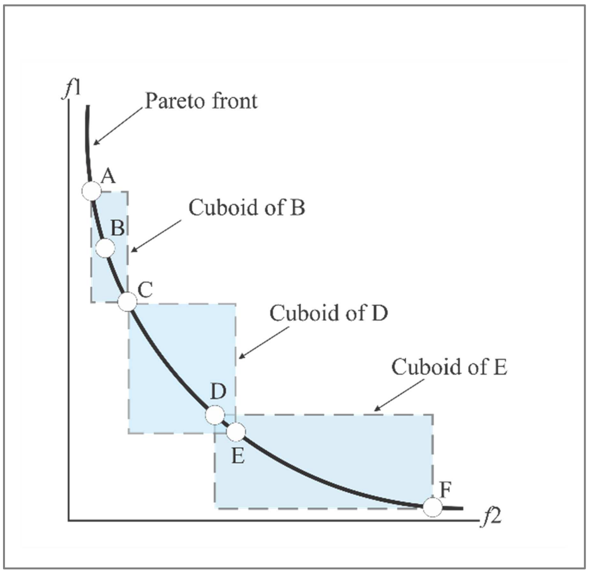

Fig. 5 Crowding distance approach

In Fig. 5, A, B, C, D, E and F represent non-dominated solutions in the PF. The cuboids represent the crowding distance of all solutions, except the extreme solutions, namely F and A, which have an infinite crowding distance. Five out of six solutions should be selected. To maintain population diversity, solution B should be selected with any one out of solutions D and E [31]. However, based on the calculation of crowding distance, solutions D and E are selected instead of solution B because they have larger crowding distance values. In this case, the diversity of the selected solutions is not preserved and leads to poor population diversity [31].

In the FA-NDSHHMO, the epsilon-clearing (ε-clearing) strategy [27] is used instead of the crowding distance to select between solutions that have the same rank. In the ε-clearing strategy, the objective space is divided into grids based on the ε value. The solutions with a difference less than ε are discarded from the population. This helps in maintaining the diversity of poulation. If there are more than enough points to complement the new population, the Euclidean distance (ED) is used to select the individual with minimum distance to Z. The population update strategy aims to improve the search ability of the algorithm. The value of ε in the ε-clearing strategy allows the decision-maker to control the density of the obtained non-dominated solutions [32]. The proposed population update strategy is integrated with the NDSHHMO algorithm to produce a non-dominated sorting Harris's hawk multi-objective optimizer based on the flush-and-ambush tactic (FA-NDSHHMO). The main steps of the FA-NDSHHMO algorithm are shown in Fig 6.

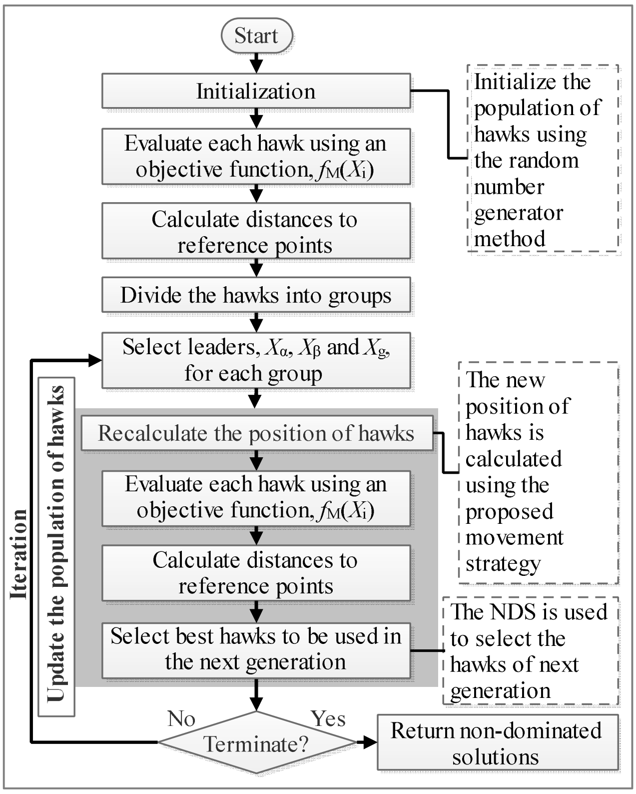

Fig. 6 FA-NDSHHMO algorithm

In the FA-NDSHHMO algorithm, the population of hawks is initialized using the random number generator method. The objective function, $f_m(X_i)$ is used to evaluate each hawk in the population. The normalized ED [27] from each solution in the objective space to Z is calculated and used as a fitness value for each hawk. This is followed by dividing the population into groups according to the number of reference points. The new generation of hawks is calculated using the proposed population update strategy, which includes two stages. In the first stage, the position of hawks is updated using the proposed movement strategy. In the second stage, the best hawks are selected by using the NDS approach. The algorithm stops when the terminate condition is met, in which t becomes equal to the maximum number of iterations (*MaxIteration*). Fig. 7 shows the selection procedure.

Algorithm 1: *Selection*

Combine P_t and Q_t to generate R_t
For each point
  Calculate the ED, between $f_m(X_i)$ and Z
end for
Performing NDS to produce the front levels.
Select individual to produce next generation:
  If number of individuals in the current front > N, perform ε-clearing strategy.
    i. Not enough individuals, move to the next front.
    ii. There is more than enough individuals, chooses the ones with the minimum ED.
  end if

Fig. 7 Main steps of *selection* procedure

In the *selection* procedure, the populations P_t and Q_t are combined to produce R, where the size of R_t is $2N$. This is followed by calculating the ED between each solution in the objective space, $f_m(X_i)$, and Z. Then, the best hawks are selected by performing the NDS with ε-clearing strategy to produce a population of size N. The new positions are evaluated using the objective function and the new leaders are selected from the new population based on the shortest ED to the Z. Fig. 8 shows the procedure of *selecting leaders*.

Algorithm 3: *Select_Leaders*

For each hawk in a group
  Calculate the ED of solution in $f_m(X_i)$ and Z
end for
Sort the EDs
Find minimum first three values to be X_α, X_β and X_δ δ, respectively.

Fig. 8 Main steps of *Select_Leaders* procedure

III. Results and Discussion

A. Experimental Design

The UF series proposed by Zhang, et al. [33] has been used in evaluating the performance of the FA-NDSHHMO. This set includes UF1-UF10 MOPs that have convex, concave and disconnected PF characteristics. The UF1-UF7 involves two objectives and UF8-UF10 are three objectives problems. These MOPs are widely used in the literature for the validation of the proposed algorithms [34-38]. This is due to the difficulties regarding convergence and diversity. Table I shows the characteristics of the UF problems.

TABLE I
Characteristics of the UF Problems

MOP	Pareto border	Dimension	*M*
UF1	Convex	30	2
UF2			
UF3			
UF4	Concave		
UF5	Discontinuous		
UF6	Discontinuous		
UF7	Convex & concave		
UF8	Concave		3
UF9			
UF10			

The performances of NDSHHMO, FA-NDSHHMO, MOGWO and MOGOA algorithms were compared under the same conditions. The population size is set at 100 individuals for each problem. The *MaxIteration* has been set at 3000. Each algorithm runs independently 10 times and the algorithm stops when the number of iterations of each running reaches the value of *MaxIteration*. The parameters for the MOGWO and MOGOA algorithms are set as recommended by their respective authors. For FA-NDSHHMO and NDSHHMO, different reference points were used with each test problem, as shown in Table II.

TABLE II
SETTINGS OF REFERENCE POINTS

MOP	***M***	**Reference point**
UF1		(0.5,0.4), (0,0.8), (0.8,0)
UF2		
UF3		
UF4	2	(0.7, 0.7); (0,0.95); (0.9, 0)
UF5		(0.6,0.6); (-0.1,0.81); (0.81,-0.1)
UF6		(0.95,0.15); (0.25,0.65)
UF7		(0.55,0.45); (0.1,0.8); (0.8, 0.1)
UF8		(0.2,0.2,0.9); (0.5,0.5,0.5)
UF9	3	(0.7,0.2,0.2); (0.2,0.7,0.2)
UF10		(0.2,0.2,0.9); (0.5,0.5,0.5)

In this study, the effectiveness of the FA-NDSHHMO algorithm is measured by the spacing (SP), inverted generational distance (IGD) and maximum spread (MS) metrics [9]. These metrics are given in Equations (8-10), respectively.

$$IGD = \frac{\sqrt[p]{\sum_{i=1}^{n} d_i^2}}{n} \tag{9}$$

where n is the number of solutions in the true PF, p = 2 and d_i^2 is the minimum ED between point i and the nearest point of the true PF. A smaller value for this metric indicates a better result, IGD=0 means that all the generated elements are in the true PF of the problem.

$$SP = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{d}-d_i)^2} \tag{10}$$

where $d_i = \min_j\left(\left|f_1^i(\vec{x}) - f_1^j(\vec{x})\right| + \left|f_2^i(\vec{x}) - f_2^j(\vec{x})\right|\right)$ for all i, j =1,2,3,…,n and $\bar{d}$ is the average of all d_i.

$$MS = \sqrt{\sum_{i=1}^{M}\max(d(a_i, b_i))^2} \tag{11}$$

where a_i and b_i are the maximum and minimum values in the i^{th} objective and M is the number of objectives. d is the ED between a_i and b_i. Note that, the smaller value for IGD and SP metrics indicates better approximation for the PF of the problem. By contrast, the larger value of MS is better. For fair comparison, the statistical measures for 10 independent runs is calculated. These measures include the mean, standard deviation (SD), best and worst values of the IGD, SP and MS metrics.

B. Experimental Results

The credibility of the FA-NDSHHMO is evaluated by comparing it with the NDSHHMO, MOGWO [10] and MOGOA [9] algorithms in solving UF MOPs. The mean, SD, best and worst values of IGD and SP and, MS metrics were calculated and used in the comparison (refer Table III). Best results are highlighted.

TABLE III
MEAN, SD, BEST AND WORST IGD, SP AND MS VALUES OF NON-DOMINATED SOLUTIONS OF THE UF PROBLEMS

MOP	**Algorithm**	**Metric**	**Mean**	**SD**	**Best**	**Worst**
UF1	FA-NDSHHMO	IGD	**0.0003**	0.0000	0.0003	0.0004
		SP	0.0288	0.0125	0.0163	0.0562
		MS	**1.5717**	0.0639	1.4997	1.7002
	NDSHHMO	IGD	0.0003	0.0000	0.0003	0.0004
		SP	0.0198	0.0078	0.0108	0.0343
		MS	1.5348	0.0653	1.4411	1.6693
	MOGWO	IGD	0.1144	0.0195	0.0802	0.1577
		SP	0.0124	0.0054	0.0146	0.0008
		MS	0.9268	0.9327	0.0688	0.8180
	MOGOA	IGD	0.1811	0.0250	0.1430	0.1811
		SP	**0.0012**	0.0011	0.0000	0.0012
		MS	0.7270	0.1507	0.9120	0.7270
UF2	FA-NDSHHMO	IGD	**0.0003**	0.0000	0.0002	0.0003
		SP	0.0139	0.0041	0.0090	0.0219
		MS	1.4043	0.0309	1.3525	1.4496
	NDSHHMO	IGD	0.0003	0.0000	0.0002	0.0003
		SP	0.0146	0.0059	0.0071	0.0230
		MS	**1.4331**	0.0358	1.3734	1.4899
	MOGWO	IGD	0.0583	0.0074	0.0498	0.0732
		SP	0.0111	0.0095	0.0036	0.0076
		MS	0.9097	0.9104	0.0287	0.8470
	MOGOA	IGD	0.0959	0.0386	0.0488	0.0959
		SP	**0.0007**	0.0011	0.0000	0.0007
		MS	0.8845	0.0353	0.9360	0.8845
UF3	FA-NDSHHMO	IGD	0.0007	0.0001	0.0005	0.0009
		SP	0.0495	0.0359	0.0112	0.1262
		MS	1.3572	0.1441	1.1416	1.5482
	NDSHHMO	IGD	**0.0006**	0.0001	0.0005	0.0007
		SP	0.0534	0.0383	0.0130	0.1195
		MS	**1.3802**	0.1772	1.1994	1.7309
	MOGWO	IGD	0.1223	0.0107	0.1049	0.1437
		SP	0.0459	0.0486	0.0145	0.0155
		MS	0.8720	0.8744	0.0056	0.8599
	MOGOA	IGD	0.2380	0.0662	0.1682	0.2380
		SP	**0.0019**	0.0024	0.0000	0.0019
		MS	0.1100	0.7060	0.4026	0.1100
UF4	FA-NDSHHMO	IGD	**0.0003**	0.0000	0.0003	0.0003
		SP	0.0115	0.0017	0.0085	0.0140
		MS	**1.4224**	0.0081	1.4136	1.4406
	NDSHHMO	IGD	0.0004	0.0000	0.0004	0.0004
		SP	0.0099	0.0018	0.0076	0.0132
		MS	1.4197	0.0073	1.4088	1.4286
	MOGWO	IGD	0.0587	0.0005	0.0580	0.0594
		SP	0.0097	0.0086	0.0039	0.0058
		MS	0.9424	0.0009	0.9433	0.9410
	MOGOA	IGD	0.0702	0.0048	0.0639	0.0702
		SP	**0.0001**	0.0002	0.0000	0.0001
		MS	0.9050	0.0139	0.9310	0.9050
UF5	FA-NDSHHMO	IGD	**0.0881**	0.0068	0.0799	0.0991
		SP	0.0231	0.0250	0.0010	0.0667
		MS	**1.6523**	0.1019	1.4855	1.8356
	NDSHHMO	IGD	0.0935	0.0147	0.0780	0.1177
		SP	0.0260	0.0193	0.0004	0.0608
		MS	1.6314	0.1060	1.4858	1.7779
	MOGWO	IGD	0.7971	0.3786	0.4680	1.7386
		SP	0.1523	0.0878	0.1625	0.0084
		MS	0.3950	0.1749	0.6104	0.0301
	MOGOA	IGD	1.1559	0.1661	0.8978	1.1559

		SP	**0.0007**	0.0005	0.0001	0.0007
		MS	0.2379	0.1131	0.4894	0.2379
UF6	FA-NDSHHMO	IGD	**0.0009**	0.0002	0.0006	0.0014
		SP	0.0135	0.0078	0.0058	0.0287
		MS	**1.5588**	0.0548	1.4625	1.6147
	NDSHHMO	IGD	0.0009	0.0002	0.0007	0.0014
		SP	0.0183	0.0118	0.0025	0.0384
		MS	1.5574	0.0649	1.4395	1.6287
	MOGWO	IGD	0.2794	0.1045	0.1934	0.5504
		SP	0.0145	0.0111	0.0125	0.0019
		MS	0.6736	0.1232	0.8149	0.3884
	MOGOA	IGD	0.7771	0.2769	0.4939	0.7771
		SP	**0.0003**	0.0004	0.0000	0.0003
		MS	0.1294	0.4600	0.0695	0.1294
UF7	FA-NDSHHMO	IGD	**0.0002**	0.0000	0.0002	0.0003
		SP	0.0281	0.0165	0.0112	0.0675
		MS	**1.4798**	0.0619	1.4138	1.5705
	NDSHHMO	IGD	0.0002	0.0000	0.0002	0.0003
		SP	0.0142	0.0042	0.0083	0.0188
		MS	1.4374	0.0315	1.4146	1.5074
	MOGWO	IGD	0.1604	0.1391	0.0628	0.4014
		SP	0.0082	0.0055	0.0086	0.0003
		MS	0.8013	0.3087	0.9875	0.0225
	MOGOA	IGD	0.1726	0.0633	0.1150	0.1726
		SP	**0.0001**	0.0001	0.0000	0.0001
		MS	0.8460	0.0792	0.9570	0.8460
UF8	FA-NDSHHMO	IGD	0.0016	0.0001	0.0014	0.0017
		SP	0.0059	0.0034	0.0038	0.0154
		MS	1.2494	0.1334	1.0694	1.4113
	NDSHHMO	IGD	**0.0015**	0.0000	0.0015	0.0016
		SP	**0.0044**	0.0007	0.0035	0.0059
		MS	**1.2938**	0.0436	1.2507	1.3747
	MOGWO	IGD	2.0578	1.1455	0.4613	3.8789
		SP	0.0069	0.0047	0.0047	0.0037
		MS	0.4457	0.1857	0.8638	0.1886
	MOGOA	IGD	0.2805	0.0749	0.2154	0.2805
		SP	0.0175	0.0085	0.0069	0.0175
		MS	0.4417	0.1586	0.6342	0.4417
UF9	FA-NDSHHMO	IGD	**0.0016**	0.0000	0.0016	0.0016
		SP	**0.0043**	0.0006	0.0034	0.0055
		MS	1.1356	0.0065	1.1266	1.1454
	NDSHHMO	IGD	0.0016	0.0000	0.0016	0.0017
		SP	0.0043	0.0003	0.0037	0.0048
		MS	**1.1557**	0.0379	1.1241	1.2424
	MOGWO	IGD	0.1917	0.0925	0.1291	0.4479
		SP	0.0174	0.0183	0.0063	0.0065
		MS	0.8399	0.1976	0.9375	0.2875
	MOGOA	IGD	0.4885	0.1445	0.3336	0.4885
		SP	0.0234	0.0041	0.0172	0.0234
		MS	0.1635	0.6424	0.0677	0.1635
UF10	FA-NDSHHMO	IGD	**0.0018**	0.0001	0.0017	0.0019
		SP	**0.0082**	0.0109	0.0035	0.0390
		MS	1.1454	0.1698	1.0695	1.6155
	NDSHHMO	IGD	0.0024	0.0004	0.0016	0.0030
		SP	0.0168	0.0134	0.0056	0.0441
		MS	**1.2173**	0.3244	0.7850	1.6402
	MOGWO	IGD	3.5945	3.4883	1.0431× 4	12.956
		SP	0.0252	0.0150	0.0154	0.0000
		MS	0.2972	0.3465	0.9283	0.0319
	MOGOA	IGD	NA	NA	NA	NA
		SP	NA	NA	NA	NA
		MS	NA	NA	NA	NA

NA: not available

The FA-NDSHHMO algorithm has the lowest mean IGD values in solving eight out of 10 problems which represents 80% of the problems. The lowest IGD value indicates that the FA-NDSHHMO algorithm has better convergence toward the true PF. The second-best mean IGD values were achieved by the NDSHHMO algorithm in solving 20% of the problems. In terms of distribution of the obtained solutions, which has been measured by using SP and MS metrics, the MOGOA produced the lowest mean SP in solving seven problems. Thus, the MOGOA has managed to achieve the lowest mean SP values in solving 70% of the problems. The second-best SP value was obtained by FA-NDSHHMO in solving 20% of the problems. However, based on the mean MS values, the FA-NDSHHMO algorithm showed better performance in solving 50% of the problems, while the NDSHHMO algorithm showed better performance in solving the other half of the problems. The pair-wise comparison between the FA-NDSHHMO and other algorithms, based on the mean IGD, SP and MS metrics, are presented in Table IV. The sign (-) denotes that the other algorithms yielded statistically better results than the FA-NDSHHMO algorithm. The sign (+) denotes cases where the FA-NDSHHMO algorithm outperforms the other algorithms.

TABLE IV
SUMMARY OF THE MEAN IGD, SP AND MS VALUES OF THE OBTAINED SOLUTIONS, FOR FA-NDSHHMO, NDSAHHMO, MOGWO AND MOGOA ALGORITHMS, IN SOLVING UF PROBLEMS

MOP	Algorithm	FA-NDSHHMO		
		IGD	SP	MS
UF1	NDSHHMO	+	+	+
	MOGWO	+	+	+
	MOGOA	+	-	+
UF2	NDSHHMO	+	+	-
	MOGWO	+	+	+
	MOGOA	+	-	+
UF3	NDSHHMO	-	+	-
	MOGWO	+	+	+
	MOGOA	+	-	+
UF4	NDSHHMO	+	+	+
	MOGWO	+	+	+
	MOGOA	+	-	+
UF5	NDSHHMO	+	+	+
	MOGWO	+	+	+
	MOGOA	+	-	+
UF6	NDSHHMO	+	+	+
	MOGWO	+	+	+
	MOGOA	+	-	+
UF7	NDSHHMO	+	+	+
	MOGWO	+	+	+
	MOGOA	+	-	+
UF8	NDSHHMO	-	-	-
	MOGWO	+	+	+
	MOGOA	+	+	+
UF9	NDSHHMO	+	+	-
	MOGWO	+	+	+
	MOGOA	+	+	+
UF10	NDSHHMO	+	+	-
	MOGWO	+	+	+
	MOGOA	+	+	+

Summarizing the results presented in Table IV, the FA-NDSHHMO algorithm outperformed the NDSHHMO, MOGWO, and MOGOA algorithms in solving most of the MOPs. This is due to the proposed population update strategy, which helps in preserving the diversity and improves the convergence of the obtained solutions.

In general terms, the results obtained from the experiments that have been carried out indicate that the use of the proposed population update strategy can significantly preserve the convergence ability towards the True PF and diversity of obtained solutions. The results emphasize that the population update strategy has the advantage in solving problems with convex, concave and discontinuous PF.

IV. Conclusion

This paper proposed the FA-NDSHHMO algorithm to solve the limitations of NDSHHMO. In the FA-NDSHHMO algorithm a new population update strategy is proposed to improve the algorithm's ability in searching. This is achieved by maintaining the population diversity and improving the convergence toward the True PF. The proposed population update strategy consists of an FA movement strategy and NDS approach. The main advantages of this strategy involve updating the population with respect to the experience of all hawks. Experimental results indicate that the FA-NDSHHMO provides superior performance compared to other algorithms, which implies the effectiveness of the proposed population update strategy. The proposed algorithm is expected to be used to solve other problems with three objectives or more and real-world MOPs.

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