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Estimation of Spatial Autoregressive Model to Demonstrate the Spatial Dependence of Cancer Patients

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Abstract—Recent years have seen a rise in interest among statisticians in spatial data analysis, which is unsurprising given the detrimental effects on results and information loss that can occur from ignoring the spatial dimension in statistical analyses. Because the components of the phenomenon under study are spatially dependent, researchers have utilized spatial regression models to examine the impact of the explanatory variables on the dependent variable. The simulation method is used when challenges or difficulties are complex to address numerically. It entails creating a system for the actual model and then performing experiments on this model. Consequently, the researchers estimated the spatial autoregressive model (SAR) using the maximum likelihood (MLE) method. The SAR included the researchers' proposed weight matrix, which was built using the Rock adjacency criterion and the Euclidean distance, as well as a modified spatial weight matrix built using the Rock adjacency criterion. The suggested weight matrix was appropriate for model estimation after comparing the spatial weight matrices using the mean absolute relative error (MAPE) criterion. By utilizing the maximum likelihood approach (MLE) with the modified spatial weight matrix and the proposed spatial weight matrix, we were able to examine the relationship between the dependent variable, which represents the number of patients in each governorate, and the selected explanatory variables, which include tumor size, age average, and the number of areas contaminated with uranium in the governorate. This study utilized the spatial model to examine real-world cancer data. Based on the pollution levels caused by uranium and other pollutants from oil refineries and other industries, the governorates of Basra and Baghdad had the highest number of cancer patients, indicating a clear spatial dependence between the location of the governorate and the increase in cancer cases.

Keywords—Spatial Autoregressive Model (SAR); spatial weight matrix; Maximum Likelihood Estimation (MLE); cancer prevalence.

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I. INTRODUCTION

Spatial econometrics is one of the most essential branches of econometrics, and its importance comes from the fact that it deals with spatial data that are characterized by containing the characteristics of spatial dependence (spatial correlation) and the characteristics of spatial heterogeneity [1]. The researchers have discussed this matter regarding two factors that make it inappropriate to deal with spatial data using traditional econometric techniques. The models are concerned with the dependence among observations during a specific period without considering spatial dependence. This leads to inefficient estimates due to the failure to achieve the analysis hypotheses and the neglect of much spatial information related to the data. It is because of the inability to employ and benefit from it when using non-spatial econometric models by [2].

In 2019, Rüttenauer [3] has presented "Spatial Regression Models," where the researchers compared spatial regression models using Monte Carlo experiments. The researchers also used several types of simulations. The researchers concluded through the Monte Carlo results that the spatial autoregressive model SAR and the spatial error model SEM contain many defects in the applied experiments. In contrast, the spatial Durbin model SDM was more flexible than them. Previous studies by [4] have presented the impact of economic, spatial, educational and material factors on the industrial development of 26 cities in China to reach the main factor that makes the industrial development of some cities faster than others and the factor that causes the concentration of highly skilled expertise in some cities. As for the spatial weight matrix, the spatial adjacency matrix was used, which depends on the adjacent borders between cities and the spatial distance-common border matrix, in which each weight was built based on the length of the common border between the two neighboring cities and the distance between the centers of the two cities.

II. MATERIALS AND METHODS

A. Spatial Dependence

When two sets of sample data are spatially dependent, an observation at point (i) depends on an observation at point (j). The following equation explains this case [5]:

$$Y_i = f(Y_j),\ \ i = 1,2,\dots,L \ \ i \neq j \dots\dots \tag{1}$$

Since the process of data collecting is related to spatial units such as cities, counties, and postal codes, it is reasonable to anticipate that sample data obtained in one area will depend on values recorded in other locations [6]. Another reason is that any value of L can be used in the above formula so that the dependence can be between several observations. Measurement mistakes in nearby spatial units might cause this, as could a lack of accuracy in the administrative boundaries of the data acquired from the sample, not reflecting the true nature of the underlying process. Secondly, the spatial component of socio-demographic, regional, or economic activity, which can be the most crucial part of the modeling issue, makes us anticipate the presence of data reliance.

B. Spatial Heterogeneity

The term spatial heterogeneity indicates the variation in relationships within the sample space, and the researchers must take into consideration that in most general cases it is possible to expect different relationships between observations to include all points in the sample space, and linear relationships can be expressed in general through the following formula [7]:

$$Y_i = X_i B_i + u_i \tag{2}$$

i: denotes the location of the observations collected at any location in the sample space where$(i = 1,2,\dots,L\ )$. X_i: vector of explanatory variables (1xk). B_i: vector of parameters with dimensions (Kx1). Y_i: vector of the dependent variable at location (observation) i. u_i: vector of random error in the linear relationship [8]. A slightly more complicated way to express this idea is to allow the function f(.) in the form (2) to vary with the position of observation i as follows [9].

$$Y_i = f(X_i B_i + u_i) \tag{3}$$

By looking at formula (3), the researchers believe that it is not possible to estimate n of the vector parameters $B_i$by giving a sample of n observation data, because there is not enough information about the entire sample data to estimate every point in the sample space, this phenomenon is referred to as the problem of "degrees of freedom" [10]. To continue the analysis, the researchers must provide characteristics or conditions for variation across space. These conditions must be scarce so that the researchers can estimate more than a few parameters.

C. Spatial Weight Matrix

The spatial relationships between observations are entered into the spatial model through the spatial weight matrix, also called the "spatial correlation matrix, " symbolized by the symbol W. It is a positive matrix, its main diagonal elements are zeros, and its dimensions are (n x n), where n represents the number of observations used in the spatial model, and the general formula for the spatial weight matrix can be written as follows [11]:

$$w = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}$$

There are many spatial weight matrices due to the different methods used in constructing them [12]. We will mention the following:

D. Spatial Contiguity Matrices

It is one of the spatial weight matrices built based on the contiguity criterion between spatial units. If the two spatial units are contiguous, they are given the value (1). However, they are given the value (0) if they are not contiguous. Likewise, if the spatial unit is not contiguous, it is given the value (0) in the spatial weight matrix when built. There are several criteria for building the spatial contiguity matrix [1] The researchers used Rock's contiguity criterion to construct the spatial contiguity matrix for this research. The example and hypothetical figure below explain how Rock's criterion was used [13].

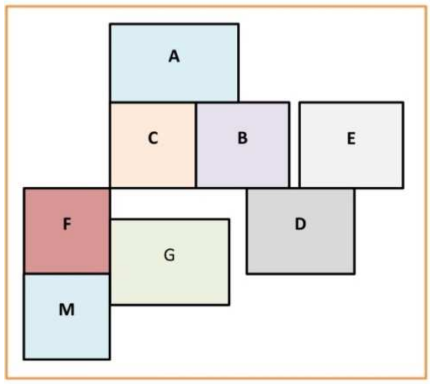

Fig. 1. A diagram showing hypothetical locations of spatial units

The figure above shows eight hypothetical locations of spatial units that share certain boundaries and points. The following spatial adjacency matrix was constructed by observing the typical boundaries that each spatial unit has with the other units. We noticed from Figure 1 that spatial unit A has a common boundary with spatial units BC and B but does not have a common boundary with the rest of the spatial units. Therefore, the WR adjacency matrix's first row (A) was built in the following way.

The spatial unit (A) that is in the neighborhood with the spatial units C and B was given the spatial value 1 ($W^R{}_{13}$, $W^R{}_{12}$=1) while the rest of the spatial units (E, F, G, D, M) in the first row do not have a common boundary with the spatial unit A and therefore were given the value 0 ($W^R{}_{1j}$=0) and also that the spatial unit A is not adjacent to itself and hence takes the spatial value 0 ($W^R{}_{11}$=0), and the rest of the rows for the other spatial units in the adjacency matrix (4) were built in the same way [14].

$$W^R = \begin{array}{c} . \\ A \\ B \\ C \\ E \\ F \\ G \\ D \\ M \end{array} \begin{array}{c} \begin{array}{cccccccc} A & B & C & E & F & G & D & M \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \tag{4}$$

E. Adjust Weights Matrix

This matrix was called the adjusted matrix because it consists of the spatial adjacency matrix, but after performing some operations on it so that the sum of each row in it equals the correct one, that is:

$$\sum_{j=1}^{n} W_{ij}^{Adj} = 1$$

The modified weight matrix is constructed by applying the following formula to the values of the adjacent matrix weights.

$$W_{ij}^{Adj} = \begin{cases} \frac{W_{ij}}{\sum W_{ij}} & if \quad W_{ij} = 1 \\ 0 & if \quad W_{ij} = 0 \end{cases} \tag{5}$$

F. Proposed Matrix

The matrix proposed by the researchers was built based on the proximity factor and the distance between the spatial units, as the spatial weights will consist of the distance between the centers of the adjacent spatial units (governorates) according to Rock's criterion for proximity. The spatial weights of the proposed matrix were built according to the following formula [15], [16]:

$$\mathrm{w}_{ij}{}^{Rd} = \begin{cases} \frac{\frac{1}{1+d_{ij}}}{\sum_{i=1}^{n}(\frac{1}{1+d_{ij}})} \text{ if } \quad \mathrm{i} \neq \mathrm{j} \end{cases} \tag{6}$$

Where:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{7}$$

where d_{ij} Euclidean distance, x_i Longitude coordinate of the first spatial unit I, x_j Longitude coordinate of the second spatial unit j, y_i Latitude coordinates of the first spatial unit I, y_j Latitude coordinate of the second spatial unit j .

G. Spatial Autoregressive Model (SAR)

Anselin [17] has proposed the general spatial autoregressive model (SAC), of which this model is a subset known as the Mixed Spatial Autoregressive Model. This is the mathematical expression of the spatial autoregressive model.

$$\underline{y} = \lambda W \underline{y} + X\underline{B} + \underline{u} \tag{8}$$

The dependent variable, y, is represented by a vector with dimensions (nx1), the spatial dependence parameter, λ, the explanatory variables, X, the error vector, u, is a normally distributed vector with dimensions (nx1), a mean of zero, and a variance of σ^2 I_n. W, the spatial weight matrix, is fixed and predetermined, and it has dimensions (nxn) [18].

H. Maximum Likelihood Method (MLE)

Ord [19] was the first to employ the maximum likelihood technique when estimating the spatial autoregressive model. This approach is essential for estimation since it provides the most accurate estimates of the model parameters out of all the available options. We updated the model in formula (8) to estimate the parameters since the least squares approach, which is used in the estimation of spatial regression models, results in biased and inconsistent estimates, which render the model inefficient.

$$\underline{\mathrm{u}} = \underline{\mathrm{y}} - \lambda \mathrm{W}\underline{\mathrm{y}} - \mathrm{X}\underline{\mathrm{B}} \tag{9}$$

$$\underline{\mathrm{u}} = (\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - \mathrm{X}\underline{\mathrm{B}} \tag{10}$$

The formula for the maximum likelihood function of the spatial autoregressive model is as follows [20]:

$$\mathrm{L}(\underline{\mathrm{B}}, \sigma^2, \lambda) = (2\pi\,\sigma^2)^{\frac{-n}{2}} + |\,\mathrm{I} - \lambda\mathrm{W}|\exp\left((1/2\sigma^2)\underline{\mathrm{u}}'\underline{\mathrm{u}}\right) \tag{11}$$

where

$$\underline{\mathrm{u}}'\underline{\mathrm{u}} = \left((\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - \mathrm{X}\underline{\mathrm{B}}\right)' \left((\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - \mathrm{X}\underline{\mathrm{B}}\right)$$

$$\underline{\mathrm{u}}'\underline{\mathrm{u}} = \left(\underline{y}'(\mathrm{I} - \lambda\mathrm{W})' - \underline{\mathrm{B}}'X'\right) \left((\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - \mathrm{X}\underline{\mathrm{B}}\right)$$

$$\underline{\mathrm{u}}'\underline{\mathrm{u}} = \underline{y}'(\mathrm{I} - \lambda\mathrm{W})'^{(\mathrm{I}-\lambda\mathrm{W})\underline{\mathrm{y}}} - \underline{y}'(\mathrm{I} - \lambda\mathrm{W})'\mathrm{X}\underline{\mathrm{B}} - \underline{\mathrm{B}}'X'^{(\mathrm{I}-\lambda\mathrm{W})\underline{\mathrm{y}}} + \underline{\mathrm{B}}'X'\mathrm{X}\underline{\mathrm{B}}$$

$$\underline{\mathrm{u}}'\underline{\mathrm{u}} = \underline{y}'(\mathrm{I} - \lambda\mathrm{W})'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - 2\underline{\mathrm{B}}'X'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} + \underline{\mathrm{B}}'X'\mathrm{X}\underline{\mathrm{B}}$$

To simplify the maximum likelihood function, we will take the natural logarithm of the formula (10) to obtain the following formula [21]:

$$\mathrm{LnL}(\underline{\mathrm{B}}, \sigma^2, \lambda) = \frac{-\mathrm{n}}{2}\mathrm{Ln}2\pi - \frac{\mathrm{n}}{2}\mathrm{Ln}\,\sigma^2 + \mathrm{Ln}\,|\mathrm{I} - \lambda\mathrm{W}| - \left(\frac{1}{2\sigma^2}\right)\underline{y}'(\mathrm{I} - \lambda\mathrm{W})'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} - 2\underline{\mathrm{B}}'X'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} + \underline{\mathrm{B}}'X'\mathrm{X}\underline{\mathrm{B}} \tag{12}$$

To estimate the model parameters, we derive the maximum likelihood function in formula (12) as follows:

$$\frac{\partial \mathrm{LnL}(\underline{\mathrm{B}}, \sigma^2, \lambda)}{\partial \underline{\mathrm{B}}} = -\left(\frac{1}{2\sigma^2}\right) 2X'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} + 2X'\mathrm{X}\,\underline{\widehat{\mathrm{B}}}_{MLE} \tag{13}$$

By making formula (13) equal to zero and simplifying it, we get [22]:

$$\underline{\widehat{\mathrm{B}}}_{MLE} = (X'\mathrm{X})^{-1}X'(\mathrm{I} - \lambda\mathrm{W})\underline{\mathrm{y}} \tag{14}$$

$$\widehat{\sigma^2}_{MLE} = \frac{\underline{\mathrm{u}}'\underline{\mathrm{u}}}{n} \tag{15}$$

$$\widehat{\sigma^2}_{MLE} = (\underline{\mathrm{y}} - \lambda\mathrm{W}\underline{\mathrm{y}} - \mathrm{X}\,\underline{\widehat{\mathrm{B}}}_{MLE})'(\underline{\mathrm{y}} - \lambda\mathrm{W}\underline{\mathrm{y}} - \mathrm{X}\,\underline{\widehat{\mathrm{B}}}_{MLE})/n \tag{16}$$

It is noted in the above formulas that it is not possible to find all of the model parameters $\underline{\widehat{\mathrm{B}}}_{MLE}$, $\widehat{\sigma^2}_{MLE}$ if the spatial dependence parameter λ is unknown. Therefore, Ord proposed a formula for calculating the determinant $|\mathrm{I} - \lambda\mathrm{W}|$as follows [23]:

$$|\mathrm{I} - \lambda\mathrm{W}| = \prod_{i=1}^{n}(1 - \lambda\,\omega_i) \tag{17}$$

$$\mathrm{Ln}\,|\mathrm{I} - \lambda\mathrm{W}| = \sum_{i=1}^{n}\mathrm{Ln}\,(1 - \lambda\omega_i) \tag{18}$$

where ω_i are the eigenvalues of the spatial weight matrix W.

A nonlinear function called the concentrated likelihood function is obtained by plugging the values of the following formulas—14, 16, and 18—into the formula of the maximum likelihood function (12). Formula 19 follows the iterative approaches of the concentrated likelihood function to obtain the spatial dependency parameter λ for the spatial autoregressive model (SAR) [24].

$$Lf = \frac{-n}{2} Ln\left[\frac{(e_o - \lambda e_L)'((e_o - \lambda e_L)}{n}\right] + \sum_{i=1}^{n} Ln\,(1 - \lambda\omega_i) \quad (19)$$

$$\underline{e_o} = \underline{y} - X\underline{\widehat{B}_o}$$

$$\underline{e_L} = W\underline{y} - X\underline{\widehat{B}}_L$$

$$\underline{\widehat{B}}_o = (X'X)^{-1}\, X'y$$

$$\underline{\widehat{B}}_L = (X'X)^{-1}\, X'Wy$$

where $\underline{\widehat{B}}_o$ vector of parameters of the regression model y on X, $\underline{\widehat{B}}_L$ vector of parameters of the regression model Wy on X, $\underline{e_o}$ vector of residuals of the regression model y on X, $\underline{e_L}$ vector of residuals of the regression model Wy on X.

I. *Moran's coefficient test*

Moran's coefficient is a statistical tool that may be utilized to quantify the degree of similarity and geographical dependence in the data of the phenomenon that will be investigated. One approach to describe it uses the I_Mc method, analogous to the Durbin-Watson test for time series data (20). It is noteworthy to observe that as the coefficient comes closer to 1, the data spreads out, and as it gets closer to 0, the data is geographically dispersed randomly [25]. This is, although the value of Moran's coefficient can take on a range that extends from 1+ to 1. To calculate Moran's coefficient, the following formula is used [26]:

$$I_{MC} = \frac{n(u'w\,u)}{S_\Theta(u'u)} \quad (20)$$

$$S_\Theta = \sum_{i=1}^{n}\sum_{j=1}^{n} Wij$$

Let S_Θ represent the sum of the elements in the matrix W, where n represents the sample size, w represents the weight matrix (adjacencies) with dimensions of n x n, and u represents the error vector (residuals) with dimensions of n x 1. Since it agrees with the conventional normal distribution, researchers [27], [28] arrived at the asymptotic distribution of Moran's statistic. Furthermore, the Z test for Moran may be carried out by utilizing the following formula [29]:

$$Z_{I_{MC}} = \frac{I_{MC} - E(I_{MC})}{\sqrt{V(I)}} \quad (21)$$

$$E(I_{MC}) = E(I_{MC}) = \frac{tr\,(M\;\;W)}{n-k}$$

$$V(I_{MC}) = \frac{tr(MWMW') + tr(MW)^2 + (tr(MW))^2}{(n-k)(n-k+1)} - \left(E(I_{MC})\right)^2$$

$$M = (I_n - X(X'X)^{-1}X')$$

where, tr sum of the elements of the main diagonal, k represents the number of explanatory variables, M represents a square and symmetrical null matrix [30], [31].

J. *Mean Absolute Percentage Error Criteria (MAPE)*

The MAPE criterion compares the methods of estimating the parameters of the spatial model and was calculated using the following mathematical formula [32], [33].

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{yi}\right| \quad (22)$$

Estimating the smallest value will be the best model parameter estimation method [34].

III. Results and Discussion

A. *Simulation technique*

By employing various techniques, the simulation approach is a procedure that involves collecting samples from a hypothetical community that is analogous to the actual community [35]. This approach is distinguished by the frequency with which it employs repetition in the random analysis of the model being investigated, followed by comparing the outcomes of the repetition to arrive at findings that may be generalized. By estimating the spatial autoregressive model with the maximum likelihood method using the modified rock adjacency matrix first, and then using the proposed adjacency matrix to show the extent of the impact of each matrix through the mean absolute percentage error (MAPE) criterion, the simulation technique was utilized to compare the modified rock adjacency matrix with the proposed adjacency matrix to arrive at the best matrix to measure the extent of the impact of adjacency between regions [36], [37]. This was accomplished by comparing the proposed adjacency matrix with the modified rock adjacency matrix with the proposed adjacency matrix.

In this direction, simulation experiments were carried out. There were four distinct sample sizes (45, 90, and 150), and the values of the dependent variable y were produced for each case. The estimation of the spatial autoregressive model (SAR) was performed by employing the maximum likelihood approach (MLE) in conjunction with the modified spatial weight matrix (w^RAdj) as indicated in Tables (1) to (3), the matrix that the researchers proposed (w^Rd) is available for viewing.

TABLE I

The values of the dependent variable y generated using simulation and the estimated values (y)^using the MLE method in light of the modified spatial

T	y	$\hat{y}\,M$ w^{RAdj}	$\hat{y}\,M$ w^{Rd}	T	y	$\hat{y}\,M$ w^{RAdj}	$\hat{y}\,M$ w^{Rd}	T	Y	$\hat{y}\,M$ w^{RAdj}	$\hat{y}\,M$ w^{Rd}
1	96	94	90	17	86	86	88	33	88	90	99
2	91	90	92	18	98	97	92	34	102	102	97
3	79	79	81	19	70	69	78	35	87	85	87
4	67	68	81	20	92	92	75	36	94	93	105
5	88	89	97	21	95	95	81	37	93	95	92
6	79	78	80	22	93	93	53	38	98	99	89
7	96	97	83	23	98	97	78	39	79	80	89
8	95	95	100	24	89	90	86	40	89	90	73
9	105	103	106	25	92	93	96	41	91	92	90
10	93	94	89	26	95	94	94	42	80	80	89

T	y	$\hat{y}\ M$ W^RAdj	$\hat{y}\ M$ W^Rd	T	y	$\hat{y}\ M$ W^RAdj	$\hat{y}\ M$ W^Rd	T	Y	$\hat{y}\ M$ W^RAdj	$\hat{y}\ M$ W^Rd
11	80	80	83	27	84	85	103	43	80	80	87
12	89	89	77	28	82	82	92	44	102	102	90
13	81	81	100	29	84	85	83	45	87	88	96
14	88	89	79	30	77	77	100				
15	87	89	95	31	87	87	84				
16	89	89	86	32	91	91	89				

TABLE II

THE VALUES OF THE DEPENDENT VARIABLE Y GENERATED USING SIMULATION AND THE ESTIMATED VALUES (Y)^USING THE MLE METHOD IN LIGHT OF THE MODIFIED SPATIAL WEIGHT MATRIX ROC (W^RADJ) AND THE PROPOSED SPATIAL MATRIX (W^RD) WHEN THE SAMPLE SIZE IS 90.

T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd
1	99	99	89	24	81	81	88	47	90	91	104	70	86	87	$\hat{y}_m$
2	77	76	78	25	98	99	79	48	96	95	85	71	93	94	90
3	88	88	82	26	98	98	92	49	89	87	74	72	92	92	91
4	82	81	82	27	79	78	87	50	87	88	82	73	90	90	87
5	94	97	79	28	97	95	73	51	84	83	84	74	107	107	71
6	86	87	73	29	97	97	83	52	85	84	80	75	87	87	96
7	91	90	90	30	80	81	85	53	96	96	105	76	104	102	81
8	87	89	88	31	90	89	101	54	74	72	91	77	86	86	81
9	81	81	91	32	91	91	98	55	91	90	99	78	75	76	79
10	94	95	87	33	96	96	101	56	87	87	77	79	90	90	117
11	69	69	83	34	85	84	94	57	91	92	71	80	78	78	93
12	83	82	103	35	95	95	97	58	81	81	80	81	78	78	106
13	93	93	84	36	77	77	103	59	76	77	85	82	100	101	89
14	94	94	107	37	81	80	84	60	103	102	82	83	100	99	108
15	86	86	94	38	90	89	96	61	108	108	102	84	98	96	84
16	70	71	92	39	93	95	75	62	81	82	82	85	92	92	85
17	81	82	72	40	87	87	88	63	92	91	98	86	83	84	95
18	73	74	84	41	78	77	85	64	82	81	71	87	102	103	98
19	79	79	107	42	65	67	84	65	96	96	94	88	84	84	112
20	81	82	100	43	97	98	90	66	93	92	88	89	89	88	93
21	92	91	101	44	95	95	95	67	74	75	97	90	74	74	87
22	86	87	100	45	89	90	75	68	95	94	83				
23	81	82	83	46	87	86	87	69	93	93	78				

TABLE III

THE VALUES OF THE DEPENDENT VARIABLE Y GENERATED USING SIMULATION AND THE ESTIMATED VALUES (Y)^USING THE MLE METHOD IN LIGHT OF THE MODIFIED SPATIAL WEIGHT MATRIX ROC (W^RADJ) AND THE PROPOSED SPATIAL MATRIX (W^RD) WHEN THE SAMPLE SIZE IS 150.

T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd
1	111	113	91	24	97	99	82	47	88	88	82	70	117	117	82
2	87	87	94	25	83	83	84	48	113	113	84	71	88	88	84
3	87	86	91	26	83	84	87	49	101	102	87	72	94	95	87
4	94	93	81	27	90	88	89	50	84	85	89	73	102	101	89
5	94	94	74	28	104	102	92	51	90	91	92	74	94	95	92
6	94	94	79	29	85	86	79	52	82	82	79	75	90	91	79
7	85	84	71	30	90	90	107	53	84	85	107	76	90	90	107
8	93	93	84	31	79	81	81	54	77	77	81	77	82	81	81
9	83	83	86	32	89	88	98	55	95	95	98	78	96	97	98
10	71	70	80	33	96	94	96	56	92	91	96	79	88	88	96
11	77	78	87	34	86	86	96	57	80	81	96	80	109	109	96
12	83	85	87	35	97	96	86	58	84	83	86	81	88	88	86
13	91	93	90	36	68	67	91	59	90	89	91	82	92	93	91
14	92	94	105	37	90	89	98	60	82	81	98	83	93	93	98
15	91	89	92	38	95	95	87	61	90	89	87	84	92	90	87
16	75	75	69	39	81	81	69	62	85	85	69	85	89	88	69
17	91	90	88	40	77	77	62	63	94	92	62	86	95	93	62
18	83	83	86	41	98	98	93	64	79	79	93	87	96	96	93
19	89	89	77	42	106	106	89	65	91	91	89	88	90	90	89
20	105	106	98	43	99	97	73	66	76	78	73	89	90	90	73
21	97	98	92	44	97	98	101	67	76	78	101	90	96	95	101
22	80	81	92	45	91	92	74	68	85	85	74	91	105	106	74
23	80	79	86	46	88	88	99	69	81	81	99	92	74	73	99
93	82	82	90	108	86	85	86	123	93	92	90	138	90	93	97
94	94	94	82	109	90	92	94	124	90	91	88	139	93	92	91
95	81	83	88	110	77	77	95	125	98	97	74	140	94	95	83
96	88	88	78	111	112	112	93	126	107	108	71	141	87	88	98
97	88	89	84	112	84	83	73	127	86	85	90	142	87	87	88
98	92	93	88	113	72	73	78	128	89	88	100	143	108	107	84
99	82	83	95	114	100	99	97	139	99	98	106	144	82	81	98
100	80	81	103	115	90	91	76	130	94	96	70	145	91	90	98
101	92	91	92	116	108	109	92	131	78	79	94	146	89	89	99
102	100	100	88	117	83	83	83	132	76	77	93	147	88	88	94

T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd	T	y	$\hat{y}_m$ W^RAdj	$\hat{y}_m$ W^Rd
103	90	90	99	118	114	115	96	133	87	87	92	148	91	92	92
104	103	103	95	119	93	91	73	134	85	86	75	149	94	93	99
105	103	103	97	120	77	80	55	135	82	83	83	150	102	101	98
106	93	94	85	121	95	94	91	136	94	93	87				
107	100	99	86	122	106	108	90	137	91	91	98				

It is important to note that the outcomes of the maximum likelihood approach MLE, which was used to estimate the value of the dependent variable ŷ, are fascinating. It is nearly impossible to distinguish them from the value produced by the simulation y. This can be seen in the graphs in Figure 2, which are presented in the following manner.

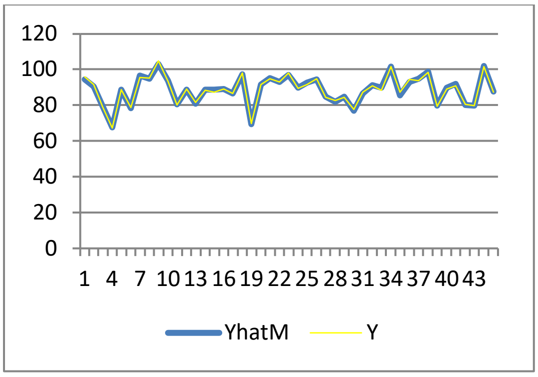

Fig. 2 Generated and the estimated values based on the modified Rock matrix, first column, and the proposed adjacency matrix, second column, for the sample set (45, 90, 150)

B. *Comparison between the Two Methods of Estimating the Parametric Spatial Autoregressive Model (SAR) According to the Presence of the Modified Spatial Weight Matrix (w^{RAdj}) and (w^{Rd})*

After the spatial autoregressive model was estimated using the maximum likelihood method (MLE), the comparison criterion (MAPE) was calculated for the parametric spatial autoregressive model (SAR) for three different sizes of data n and at three different values for each of λ and σ^2 in light of the modified spatial weight matrix (w^{RAdj}) and (w^{Rd}). The results were presented in Tables 4 and 5.

Through tables (4) and (5), and depending on the lowest value of the (MAPE) criterion for each of the modified spatial weight matrices and the proposed matrix, and for different values of the model parameters, λ σ^2 B, and for each sample size n, it was concluded that the MLE method under the modified ROC matrix is better than the proposed matrix in estimating the spatial autoregressive model (SAR)) for all sample sizes and for different values of the model parameters, ,λ σ^2 B,.

This can also be observed in Figure 3, which shows the illustrative drawing of the MAPE values for each matrix used in the estimation.

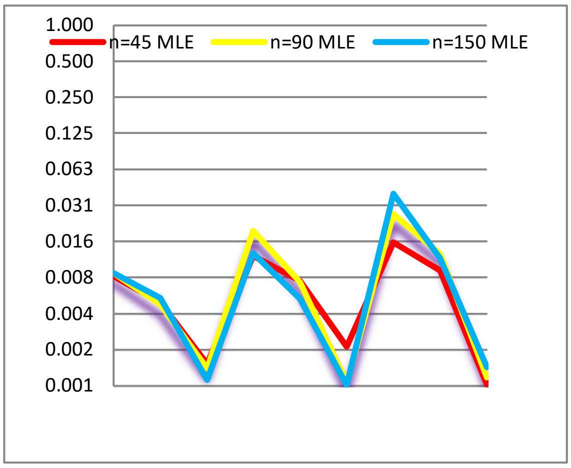

Fig. 3 MAPE criterion values for the three sample sizes n and for the MLE methods, using the two spatial weight matrices (w^{RAdj}) and ($w^{\wedge Rd}$) with B1 respectively

TABLE IV
AVERAGE RESULTS OF THE MAPE CRITERION VALUES FOR THE THREE SAMPLE SIZES N AND FOR THE MLE METHODS AND USING THE TWO SPATIAL WEIGHT MATRIX (W^{RADJ}) AND (WRD) WITH B1(79.8, 2.6, 45.7, 7.2)

B(1)		$\sigma^2$1=.5			$\sigma^2$2=1.0			$\sigma^2$3=1.5		
		λ 1=0.2	λ 2=0.5	λ 3=0.9	λ 1=0.2	λ 2=0.5	λ 3=0.9	λ 1=0.2	λ 2=0.5	λ 3=0.9
n=45	MLE W^{RADJ}	0.00694	0.00532	0.00121	0.05367	0.00418	0.00105	0.04806	0.00919	0.00217
	MLE W^{Rd}	0.01018	0.00446	0.00143	0.02160	0.00532	0.00155	0.03434	0.04505	0.00303
n=90	MLE W^{RADJ}	0.00980	0.00474	0.00143	0.01696	0.00477	0.00133	0.01206	0.00654	0.00130
	MLE W^{Rd}	0.24981	0.79708	0.94152	0.24290	0.84799	0.99751	0.70249	1.06707	1.00217
n=150	MLE W^{RADJ}	0.00852	0.00536	0.00090	0.01464	0.00530	0.001004	0.01679	0.01228	0.00096
	MLE W^{Rd}	0.00951	0.00460	0.00103	0.01313	0.00484	0.00100	0.01554	0.00635	0.00135

TABLE V
THE AVERAGE RESULTS OF THE MAPE CRITERION VALUES FOR THE THREE SAMPLE SIZES N AND FOR THE MLE METHODS AND USING THE TWO SPATIAL WEIGHT MATRIX (W^{RADJ}) AND (W^RD) WITH B2(56.7, 0.24, 1.04, 3.06)

B(2)		$\sigma^2$1=.5			$\sigma^2$2=1.0			$\sigma^2$3=1.5		
		λ 1=0.2	λ 2=0.5	λ 3=0.9	λ 1=0.2	λ 2=0.5	λ 3=0.9	λ 1=0.2	λ 2=0.5	λ 3=0.9
n=45	MLE W^{RADJ}	0.0079	0.00479	0.00146	0.01177	0.00733	0.00203	0.01533	0.00883	0.00099
	MLE W^{Rd}	0.00960	0.00479	0.00144	0.01498	0.00432	0.00173	0.01948	0.00513	0.00128
n=90	MLE W^{RADJ}	0.00845	0.00461	0.00135	0.01903	0.00708	0.00103	0.02650	0.0122	0.00113
	MLE WRd	0.03081	0.76922	0.99044	0.54527	0.87074	0.99212	0.51676	1.86773	1.00288
n=150	MLE WRADJ	0.00842	0.00520	0.00108	0.01245	0.00515	0.00098	0.03918	0.01128	0.00138
	MLE WRd	0.00813	0.00521	0.00116	0.01534	0.0055	0.00101	0.03576	0.00610	0.00103

C. *The Applied Aspect*

To study the impact of certain variables on cancer prevalence and the degree to which spatial adjacencies between the data affect the disease, a method was used to estimate the spatial autoregressive model SAR on real data representing cancer patients' records for all governorates in Iraq. The data in question is from the geographical information system (GIS), which is data about cancer spread over Iraq's eighteen governorates. The explanatory variables include average age, average tumor size, and the number of uranium-contaminated areas. The dependent variable is the number of cancer cases, and it takes into account 90 observations chosen randomly from each of Iraq's governorates, representing the most common cancer types in each. The researchers' proposed matrix relies on both the neighborhood factor and distance. The distance matrix, which was derived from the Ministry of Planning - Central Agency for Statistics, measures the distance between the centers of neighboring governorates using the coordinates of the points (x, y) unique to each governorate's center.

D. *Estimating the Spatial Autoregressive Model SAR Using the Maximum Likelihood Method MLE under the Spatial Weight Matrix (w^RAdj).*

Using the MLE method, the SAR model was calculated with an estimated value of (0.286949098) for the model's spatial dependence parameter (λ). Table 6 displays the estimated values of the dependent variable ŷ, which comprises both the real and estimated values, based on the modified Rock spatial weight matrix and the proposed matrix. Regarding the outcomes of Moran's Z_Mtest, they were 4.28324906 when the updated spatial weight matrix w^RAdj was utilized. At a significance level of 0.05, the calculated value is 1.96; however, when we compare the tabular value, which is smaller, we find that the tabular value is less yet, suggesting that the data exhibit geographical dependency. The mean absolute relative error (MAPE) for the spatial autoregressive model SAR was determined by utilizing the modified Rock spatial weight and the proposed matrix, which were found to be (1.0472) and (1.0535), respectively, after the estimated values ŷ for the dependent variable were obtained.

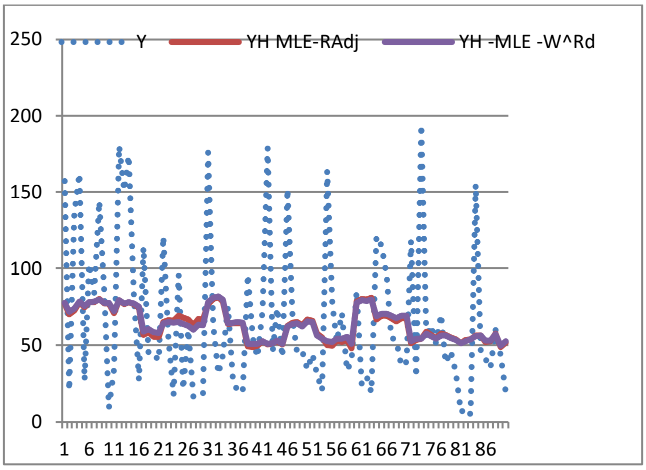

Fig. 4 Methods of estimating the spatial autoregressive model in light of the two spatial weight matrices. w^{RAdj}, w^{Rd}

TABLE VI
THE ACTUAL AND ESTIMATED VALUES OF THE DEPENDENT VARIABLE Y USING THE MAXIMUM LIKELIHOOD METHOD MLE IN LIGHT OF THE WEIGHT MATRIX W^{RADJ} AND WRD.

T	y	$\hat{y}$ w^{RAdj}	$\hat{y}$ w^{Rd}	T	y	$\hat{y}$ w^{RAdj}	$\hat{y}$ w^{Rd}	T	y	$\hat{y}$ w^{RAdj}	$\hat{y}$ w^{Rd}	T	y	$\hat{y}$ w^{RAdj}	$\hat{y}$ w^{Rd}
1	157	77	77	24	96	65	69	47	67	64	64	70	39	69	68
2	20	72	70	25	21	64	68	48	45	64	65	71	118	52	51
3	136	74	72	26	60	62	66	49	46	62	63	72	29	54	53
4	162	78	78	27	16	60	63	50	34	65	67	73	194	54	54
5	27	75	75	28	16	63	67	51	44	65	66	74	54	57	59
6	103	78	78	29	18	63	67	52	31	56	57	75	63	56	57
7	84	78	78	30	176	78	77	53	20	54	55	76	51	54	55
8	144	80	80	31	63	81	80	54	164	52	50	77	70	56	57
9	83	77	77	32	31	82	82	55	77	52	49	78	38	56	57
10	9	77	77	33	43	79	78	56	52	54	53	79	46	54	55
11	34	72	71	34	75	64	64	57	71	54	52	80	32	53	54
12	180	79	79	35	26	64	64	58	33	55	53	81	6	51	51
13	151	77	77	36	20	65	64	59	40	51	48	82	9	53	53
14	174	78	78	37	19	65	64	60	85	77	78	83	5	54	54
15	83	77	77	38	96	51	49	61	24	79	80	84	155	56	56
16	27	75	74	39	54	51	49	62	29	79	80	85	45	56	56
17	114	59	56	40	42	51	49	63	19	80	81	86	41	53	52
18	44	60	58	41	72	52	52	64	121	69	67	87	35	53	52
19	44	58	55	42	179	51	50	65	115	70	69	88	60	57	58
20	40	58	55	43	46	51	51	66	95	71	69	89	43	50	49
21	121	64	65	44	74	53	53	67	55	69	67	90	19	52	51
22	54	65	66	45	42	51	50	68	53	67	65				
23	17	65	66	46	152	62	63	69	36	69	68				

Table 6 shows the estimated values of the number of people infected with cancer based on the modified Rock matrix and the proposed matrix. There is a minimal difference between them, indicating the proposed method's effectiveness in estimating the spatial autoregressive model. The Moran coefficient also showed that the place affects the incidence of cancer due to the large number of infections in both Basra Governorate and Baghdad Governorate, in particular, as a result of the pollution of these two governorates with uranium on the one hand and the presence of oil refineries and the pollution they cause, which affects the increase in cancerous diseases.

IV. Conclusion

A modified spatial weight matrix (wRAdj) based on the Rock adjacency criterion is better than a proposed spatial weight matrix (wRd) that combines the adjacency factor using the Rock criterion and the distance factor using the Euclidean distance. This conclusion was reached after performing simulation experiments on the parametric spatial autoregressive (SAR) model and estimating the model parameters using the maximum likelihood (MLE) method based on the values of the mean absolute relative error (MAPE) criterion. The weight matrix (wRAdj) had the lowest value for the comparison criterion (MAPE). When the sample size was 45 or 90 and the parameter value was 〚(σ〛^2=1,λ=0.9), the best estimate for the spatial autoregressive model was obtained. On the other hand, when the sample size was 150 and the variance value was 〚(σ〛^2=1), the best estimate for the model could be obtained. Considering the suggested spatial weight matrix (wRd), it was determined that the optimal estimation for the spatial autoregressive model can be achieved with the following parameter values: 〚(σ〛^2=0.5,λ=0.9) for a sample size of 90, 〚(σ〛^2=1,λ=0.9) for a sample size of 150, and 〚(λ=0.9) for a sample size of 45.

The data is found to be spatially dependent when applying the spatial weight matrix (wRAdj). This was determined after estimating the spatial regression model with real data for the number of infected people distributed throughout 18 governorates in Iraq, using the findings of the Moran coefficient test. In other words, the tumor rate and the presence of uranium-rich sites determine the cancer incidence in each governorate. The highest cancer incidence was in Baghdad and Basra, followed by Najaf and Karbala, then Dhi Qar, Nineveh, and Anbar. Among the cancer types with the highest incidence, breast cancer was among the least common.

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