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Integrating Statistical Models for Forecasting Nonlinear Time Series

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***Abstract*—The issue of unemployment is one of the most prominent challenges facing the growth and development of the Iraqi economy due to its profound repercussions on the economic and social conditions. What exacerbates the severity of the unemployment problem is its prolonged existence and recurring nature, as well as its manifestation in various forms, including disguised unemployment, over recent years. It is noteworthy that unemployment is no longer limited to uneducated or moderately educated youth, but has also extended to those with higher degrees. Although the state has taken the initiative to employ youth with educational degrees, unemployment rates have not shown an apparent decrease due to the large number of unemployed people in Iraq. In this research, the best model was chosen from among the non-linear time series models to predict unemployment rates in Iraq, which included the Logistic, Gompertz, and Chapman-Richard models. It was found that the best model for predicting unemployment rates in Iraq is the Chapman-Richard model. Consequently, we suggest leveraging this research to develop future strategies that address the unemployment issue in Iraq and expand job opportunities in both the public and private sectors to meet the growing number of unemployed individuals. We propose applying the best model from this study to explore other economic and social issues. We advise enhancing efforts to accurately gather and record data so that it can be utilized by researchers in subsequent studies.**

***Keywords*—Non-linear time series model; logistic model; Gompertz model; Richard model; Chapman Richards model.**

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I. INTRODUCTION

The issue of unemployment is considered one of the most significant challenges that Iraq currently faces. This is because the high number of unemployed individuals symbolizes a waste of the energy of the youth who are jobless, in addition to the significant impact that unemployment has on the country's political, economic, and social situations [1], [2]. Despite Iraq's possession of natural, financial and agricultural resources, there is a deficiency in the possibility of exploiting those resources in a way that works on the optimal use of the energies of the youth, and the phenomenon of unemployment has worsened as a result of economic policies focused on military spending in previous periods. To accomplish this, it is necessary to have the appropriate foundations for developing a policy that ensures employment opportunities to meet the country's needs.

Additionally, it is essential to pay attention to the problem of anticipating unemployment rates [3], [4], [5]. We made use of non-linear time series models because these models made a significant contribution to the accurate description of the characteristics from which the time series of the phenomenon that was being studied is generated, as well as to the interpretation of the behavior of that phenomenon and the utilization of the results to predict the future values of that phenomenon [6], [7], [8]. By employing several nonlinear time series models, the research aims to statistically model the series of unemployment rates in Iraq, with the ultimate goal of selecting the most suitable model for forecasting unemployment rates in Iraq. Linear models are used to represent the changes that occur in time series and to determine the general trend of the series.

However, there are situations in which data follows a non-linear trend, which means that linear methods or models cannot be utilized. In such situations, we turn to other models that are suitable for the data, known as non-linear models [9]. There are numerous nonlinear time series models, such as exponential, quadratic, and logarithmic models, among others. Some of these nonlinear models are not convertible, which means that they cannot be converted into linear formulas. On the other hand, some nonlinear models are convertible, which means that they can be converted into linear formulas [10]. These linear formulas are often used because they produce accurate results [11]. This investigation utilizes four nonlinear models that can be transformed into linear models.

II. MATERIALS AND METHODS

A. Logistic Model

The model was first used by the Belgian scientist Verhuist in 1845. What distinguishes this model is that it is symmetrical around the turning point. This property indicates that the process that occurs after the turning point is the same as the process that occurred after it [12], [13]. The model is expressed according to the following formula:

$$Z_t = \frac{a}{1+\beta e^{-kt}} + e_t \quad (1)$$

Since:
Z_t: Dependent variable
t: Independent variables.
(a, β, k): Model parameters
e_t: random error
Based on the maximum likelihood (MLE) method, the model parameters were estimated in Equation (1) [14]:

$$L(Z_t) = \frac{\alpha^n}{1+\beta^n e^{-k\sum_5^m \; 1t}} + \sum_i^m \; Te_t \quad (2)$$

$$Ln[L(Z_t)] = nLn(a) - Ln\left[1 + \beta^n e^{-k\sum_y^t \; rt}\right] \quad (3)$$

Taking the derivative in equation (3) for the parameters (a, β, k), and setting it equal to zero:

$$\frac{\partial Ln[L(Z_t)]}{\partial(a,\beta,k)} = 0 \quad (4)$$

We obtain a set of equations with the number of coefficients in the model, and since these equations are nonlinear, it is challenging to estimate them using conventional methods. Therefore, we resort to using one of the iterative methods to estimate these coefficients, such as the Newton-Raphson iterative method [15], [16].

B. Gompertz Model

This model was introduced by Benjamin Gompertz in 1825 to fit mortality tables, and the Gompertz function is a sigmoid function, a type of mathematical model commonly used for time series analysis. The Gompertz model is slow at the beginning and end of the period. The right curve of the function approaches the left or lower curve of the function more closely. In contrast to the logistic function, the two variables approach the curve symmetrically. The Gompertz model is a special case of the four-parameter Richards model, and it belongs to the Richards family of three-parameter sigmoidal growth models along with the exponential models, the logistic model, and the von Bertalanffy model [17]. The general formula for the Gompertz model is:

$$Z_t = ae^{-\beta e^{-kt}} + e_t \quad (5)$$

Since: (a,β,k) represent the model parameters, and the value of the response variable (Z) changes with time (t). To estimate the parameters of the Gompertz model, the maximum likelihood (MLE) method was used as follows:

$$L(\hat{Z}_t) = a^n e^{-\beta^{n} e^{-k\sum_y^t \; yt}} \quad (6)$$

$$Ln[L(\hat{Z}_t)] = nLn(a) - \beta^{n_e -k\sum_y^m \; ut} \quad (7)$$

The model equation is still non-linear, so we take (Ln) again as follows.

$$Ln\left[Ln[L(\hat{Z}_t)]\right] = Ln[nLn(a)] - nLn(\beta) - k\sum_m^t \; ut \quad (8)$$

$$Let: Ln\left[Ln[L(Z)]\right] = \hat{Y}_t^* \;, Ln(a) = \alpha^* \;, \qquad Ln(\beta) = \beta^*$$

Substituting the hypotheses into the above equation, we get the following:

$$\hat{Z}_t^* = nLn(a^*) - n\beta^* - k\sum_m^t \; ut \quad (9)$$

We derive the model in Equation (9) for the parameters (a, β, k) and set the derivative equal to zero to obtain a set of nonlinear equations that are difficult to estimate using conventional methods. Therefore, we resort to using one of the iterative methods to estimate these parameters, such as the Newton-Raphson iterative method, which was relied upon in the research [18], [19].

C. Chapman-Richard Model

Considered universal for various nonlinear models, such as the logistic model and the Chapman-Richard model, this approach serves to provide an accurate and more realistic depiction of many events. Many research studies, including those on the elements influencing animal growth—that of fish, cows, horses, etc.—have utilized the Chapman-Richard model [20]. Selected to explain the law of the transmission of infectious diseases, this model is known by the formula:

$$Z_t = a(1 - \beta e^{-kt})^\theta + e_t \quad (10)$$

Since:
Z_t: The response variable represents.
(a, β, k): Model parameters represent
Following this formula will help one estimate the model parameters using the maximum likelihood (MLE) approach:

$$L(Z_t) = a^n \sum_u^m \; t(1 - \beta e^{-kt})^\theta + \sum_y^m \; me_t \quad (11)$$

$$Ln[L(Z_t)] = nLn(a) - \theta \sum^t \; mLn(1 - \beta e^{-kt}) + \sum_m^u \; tLn(e_t) \quad (12)$$

Since:
n: Maximum limit of (MLE).
From equation (12), we take the derivative of the parameters and set it to zero:

$$\frac{\partial Ln[L(Z_t)]}{\partial(a,\beta,k)} = 0 \quad (13)$$

We thus derive a set of equations. As these equations are nonlinear and challenging to estimate using standard approaches, we turn to using one of the iterative methods to estimate these coefficients, such as the Newton-Raphson iterative method.

D. Richard Model

This model was used by the scientist Richard [12], and the mathematical formula for the model is [21], [22], [23]:

$$Z(t) = \frac{a}{(1+\beta expexp\,(-kt))^{\frac{1}{m}}} + \; et \quad (14)$$

Since:
Z :. Response variable
t :. independent variable
a, β, k, m : Parameters to be estimated
et : Random error

The parameter estimates are found using the maximum likelihood (MLE) method for the following model [24]:

$$Z(t) = a(1 + \beta\ exp\ exp\ (-kt)\ )^{\frac{1}{m}} + et \qquad (15)$$

By entering ln into both sides of the model and taking the derivatives concerning the parameters and setting them equal to zero, we get the following [25]:

$$(1 + \beta\ exp((-kt))^{\frac{1}{m}} \qquad (16)$$

$$(-a(1 + \beta\ exp\ exp\ (-kt)\ )^{\frac{1}{k}-1}(exp\ exp\ (-kt)\ ) \qquad (17)$$

$$(a\beta t/m)\ \ (1 + \beta\ exp\ exp\ (-kt)\ )^{\frac{1}{k}-1}(exp\ exp\ (-kt)\ ) \qquad (18)$$

$$(1 + \beta\ exp\ exp\ (-kt)\ )^{\frac{1}{k}}((-kt)\ )\ m^{-2} \qquad (19)$$

We use one of the iterative methods—such as the Newton-Raphson iterative method—to estimate these coefficients since the equations above are nonlinear equations that are challenging to estimate with traditional approaches [26], [27].

E. Selecting the Ideal Model

Following one of the estimation techniques—the maximum likelihood method—the estimated parameters of the nonlinear models utilized in the research and the models' fit to their data were acquired. The subsequent step involves selecting the most suitable model from among the nonlinear models being investigated [28]. Among the many criteria that can be used to choose the best model, Bayesian Information Criteria and Akaike's Information Criteria are two examples. Based on these criteria, nonlinear models are evaluated, and the model that is deemed to be the most optimal is chosen based on the lowest value for each of these criteria [29].

F. Akanke's Information Criterion

Akaike's Information Criterion (AIC) is a standard tool for modeling time series data to measure the fit of a statistical model for M parameters. It can be written as follows.

$$AIC(M) = nLn\hat{\sigma}_{\alpha}^{2} + 2M \qquad (20)$$

Since:
M: number of model parameters.
n: number of views

The ideal rank of the model is chosen by the value M, which corresponds to the lowest value of the criterion, and the model that gives the lowest value of the AIC criterion is the best model [30].

G. Bayesian Information Criterion

One of the criteria for determining the model rank is the Bayesian information criterion (BIC) [31]. The Bayesian information criterion gives a consistent estimate of the true rank, unlike the Akaike criterion (AIC), which often gives an estimate with a higher rank than the true rank. It takes the following formula:

$$BIC(m) = nLn(\hat{\sigma}_{a}^{2}) - (n-m)Ln\left(1 - \frac{m}{n}\right)mLnn + mLn\left[\frac{1}{m}\left(\frac{\hat{\sigma}_{z}^{2}}{\hat{\sigma}_{a}^{2}} - 1\right)\right] \qquad (21)$$

Since:

$\hat{\sigma}_{a}^{2}$: the greatest possible estimator of $\hat{\sigma}_{a}^{2}$
$m.$: the number of parameters in the model
$\hat{\sigma}_{z}^{2}$: the sample variance of the time series.
The model that corresponds to the lowest value of the (BIC) criterion is the best model.

H. Root Mean Square Error

It is one of the measures of predictive power and can be written in the following formula:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \quad me_{t}^{2}}{n}} \qquad (22)$$

Since:
n: Sample size

III. Results and Discussion

A. Predictive Accuracy Metrics

When it comes to selecting the most suitable method for prediction, the most important factor to consider is the predictive accuracy. The majority of forecasts have a tendency to have a certain error rate, or, to put it another way, inaccuracy in prediction. As a result, it is necessary to conduct studies to determine the prediction accuracy of these values by investigating a selection of the measures, which are as follows:

B. Mean Squared Errors

One of the measures of predictive power, the MSE is calculated by squaring the sum of the errors and then taking the average of the squares of the errors, and dividing it by the number of observations (n) As in the following equation:

$$MSE = \frac{1}{n}\sum_{i=1}^{n} \quad me_{t}^{2} \qquad (23)$$

The best model is the one is the one that yields the lowest mean squared error.

C. Root Mean Square Error

It is the square root of the mean square error. The following formula represents it:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \quad Ee_{t}^{2}} \qquad (24)$$

The best model that gives the lowest root mean square errors.

D. Time Series Analysis

To achieve the research objective, we selected a time series of unemployment rates in Iraq for the period from 1990 to 2022 from the Central Statistical Organization/Annual Statistical Collection, and Figure (1) shows a graph of the time series of unemployment rates.

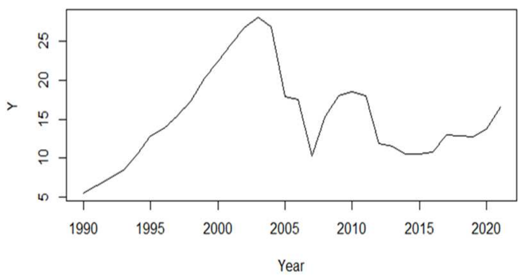

Fig. 1 Time series diagram of unemployment rates in Iraq

To test the stationarity of the time series of unemployment rates, we studied the Augmented Dickey-Fuller Test. One finds whether a time series is stationary using the Augmented Dickey-Fuller (ADF) test. At certain lag levels, the table shows ADF test results for two scenarios: (1) without trend and drift, and (2) with trend and drift. Both times, the ADF test statistic (ADF) and p-values are stated. Regarding the situation without trend and drift, the ADF values span -0.0379 to -0.3899 over lag levels 0 through 3. Since the related p-values are all over 0.5, it is impossible to refute the null hypothesis of a unit root-non-stationarity. This implies that in its natural form, the time series is probably non-stationary.

Concerning trend and drift, the ADF values are more negative, ranging from -1.87 to -2.59. The null hypothesis cannot be disproved, so the p-values remain high, between 0.611 and 0.335. Though it somewhat increases stationarity properties, the presence of a trend and drift does not render the series stationary. The results generally show that, under both criteria, the time series is non-stationary. Before using time series models like ARIMA, additional modifications, such as differencing, may be necessary to achieve stationarity.

TABLE I
AUGMENTED DICKEY-FULLER TEST

With no trend and no drift			**With trend and drift**		
Lag	**ADF**	**p-value**	**Lag**	**ADF**	**p-value**
0	-0.0379	0.626	0	-1.87	0.611
1	-0.1873	0.583	1	-2.09	0.523
2	-0.3899	0.524	2	-2.59	0.335
3	-0.1644	0.589	3	-2.31	0.438

We note from Table 1 that the time series of unemployment rates has a unit root but is not stable, as indicated by the p-value, which is greater than 0.05.

E. Application for Mann-Kendall Test

To apply non-linear time series models, the linearity of the time series of unemployment rates was tested using the Mann-Kendall Test, as shown in Table 2 and Figure 2. The p-values shown in the table correspond to various lag values in a statistical test—likely the Augmented Dickey-Fuller (ADF) test—that examines the stationarity of a time series. More substantial evidence against the null hypothesis of a unit root suggests a stationary time series by a lower p-value.

The p-values for delays 1 through 7 run from 5 × 10 − 7, 5×10 −7 to 5.4×10 – 8. Strong rejection of non-stationarity is shown by 5.4×10 −8. The p-value decreases as the latency increases, thereby strengthening stationarity at larger lags. The p-values for lags 8 through 14 keep decreasing; at lag 14, they reach 1.9 x 10 − 15. This confirms stationarity even more, as the likelihood of a unit root presence approaches zero. The consistently declining p-values imply that adding higher-order lag factors enhances the evidence of dataset stationarity.

TABLE II
MANN-KENDALL TEST

Lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**
p-value	5e-07	2.1e-09	8.5e-10	2.2e-09	7.7e-09	2.4e-08	5.4e-08
Lag	8	9	10	11	12	13	14
p-value	4.8e-08	9.8e-09	4.4e-10	8.3e-12	1.5e-13	7.7e-15	1.9e-15

The results generally indicate that the time series is likely to be stationary at all stated lag levels. The declining trend in p-values suggests that adding more lags enhances the test's capacity to detect stationarity. As such, the dataset is suitable for time series modeling, without requiring additional treatment, as shown in Figure 2.

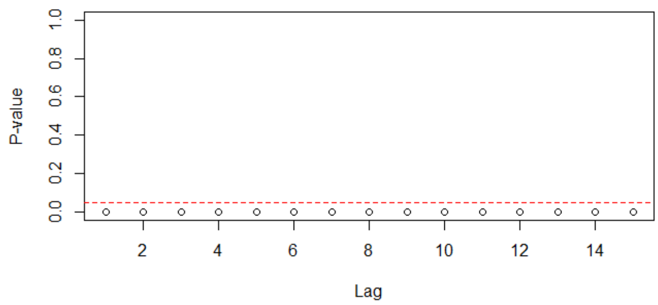

Fig. 2 Mann-Kendall test graph

We note from Table 2 that the p-values are smaller than the significance level of 0.05, which indicates the presence of a general trend in the time series, as well as the non-linearity of the series. Graph (2) confirms this statement, as it shows that the p-value falls in the zero region.

F. Homogeneity Test

To test the homogeneity of the time series data for unemployment rates, the von Neumann Test was conducted, and the p-value was found to be 2.923e-07, which is less than the significance level of 0.05, indicating heterogeneity in the time series data for unemployment rates in Iraq. An R version 3.6.1 program was used to estimate the parameters of nonlinear time series models, determining the best model for predicting unemployment rates in Iraq, as shown in Table 3.

TABLE III
ESTIMATION OF NONLINEAR TIME SERIES MODELS

	Logistic			
par	**estimate**	**sd**	**t stat**	**p-value**
α	22.935	7.0650	3.24629	*
β	0.8525	0.1475	5.77992	**
k	-	-	-	-

Table 4 presents the Gompertz model estimation findings, which are widely used in various disciplines, including population dynamics, economics, and biomedical sciences, to predict growth processes. Estimated coefficients, standard deviations (SD), t-statistics (t-stat), and p-values are among the outputs. With a standard deviation of 3.5625 and a t-statistic of 5.789474, the first expected parameter is 20.625.

The high t-statistic points to rather substantial statistical significance. Analogously, the second parameter estimate, 0.2842, indicates its significance in the model with a t-statistic of 4.874665 and a standard deviation of 0.00583. Highly significant is likewise the third parameter, 0.07164, with a standard deviation of 0.0119 and a t-statistic of 6.018203. The existence of "" in the p-value column implies that, at conventional significance levels (e.g., $p < 0.01$ or $p < 0.05$), the Gompertz model efficiently captures the fundamental trend in the data. These findings enable the model to be regarded as reliable for characterizing the observed occurrence; however, for greater robustness, further validation using other models or additional data could be advantageous.

TABLE IV
ESTIMATION OF NONLINEAR TIME SERIES FOR GOMPERTZ MODELS

Gompertz			
Estimate	**sd**	**t stat**	**p-value**
20.625	3.5625	5.789474	**
0.02842	0.00583	4.874665	**
0.07164	0.0119	6.018203	**

Similar to the first parameter estimate, the second parameter estimate, which is 0.2842, indicates that it is significant in the model with a t-statistic of 4.874665 and a standard deviation of 0.00583. With a standard deviation of 0.0119 and a t-statistic of 6.018203, the third parameter, which is 0.07164, is also highly significant when compared to the other parameters. It can be inferred from the presence of the symbol in the p-value column that, when considering traditional significance levels, such as $p < 0.01$ or $p < 0.05$. The Gompertz model can effectively capture the fundamental trend present in the data. In light of these findings, the model can be considered trustworthy for describing the observed occurrence; however, to ensure its robustness, additional validation using other models or more data may be beneficial, as shown in Table 5.

TABLE V
ESTIMATION OF NONLINEAR TIME SERIES FOR CHAPMAN-RICHARDS MODELS

Chapman Richards			
estimate	**sd**	**t stat**	**p-value**
29.154	0.8460	34.461	***
0.953	0.047	20.2766	***
0.253	0.06902	3.6656	*
0.936	0.064	14.51563	***

Therefore, the Richard model can effectively capture the fundamental trend that is present in the data. In light of these findings, the model can be considered trustworthy for describing the observed occurrence; however, to ensure its robustness, additional validation is presented in Table 6.

TABLE VI
ESTIMATION OF NONLINEAR TIME SERIES FOR RICHARD MODELS

Richard				
par	**estimate**	**sd**	**t stat**	**p-value**
α	27.851	2.1490	12.9599	***
β	0.827	0.173	4.78035	**
k	0.0554	0.0054	10.2593	***
θ	0.929	0.071	13.08451	***

Then, a comparison was made between the four models using the comparison criteria shown in Table 7.

TABLE VII
COMPARISON CRITERIA

Model	RMSE	AIC	BIC	R^2
Logistic	10.1615	189.68	194.21	0.3374
Gompertz	8.11942	178.26	182.24	0.4048
Richard	6.23688	140.84	143.80	0.5051
Chapman Richards	4.61605	136.62	139.07	0.6199

The best model for predicting unemployment rates was found to be the Chapman-Richards model, because it has the lowest value for (RMSE, AIC, BIC) and the highest value for (R^2). Therefore, the estimated equation for the Chapman-Richards model can be written according to the following formula:

$$\hat{Z}_t = 29.154(1 - 0.953e^{-0.253t})^{0.936} \qquad (25)$$

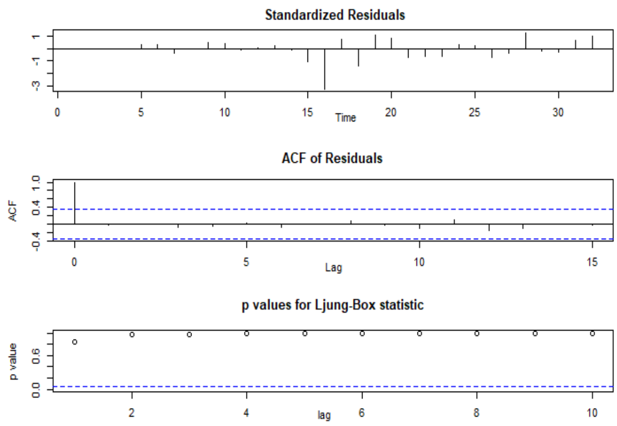

Fig. 3 Represents the Chapman-Richards model residual test

Based on Equation (25), we can derive the predictive values of unemployment rates in Iraq from 2022 to 2031, as shown in Table 5 and Figure 4. Figure (3) shows that the autocorrelation coefficients of the standard residuals fall inside the confidence limits and that the p-values of the Ljung-Box statistic are higher than the significance level of 0.05.

TABLE VIII
PREDICTIVE VALUES OF UNEMPLOYMENT RATES IN IRAQ

2022	2023	2024	2025	2026	2027	2028	2029	2030	2031
18.70	20.65	21.26	21.46	21.85	22.52	23.72	24.78	25.67	26.24

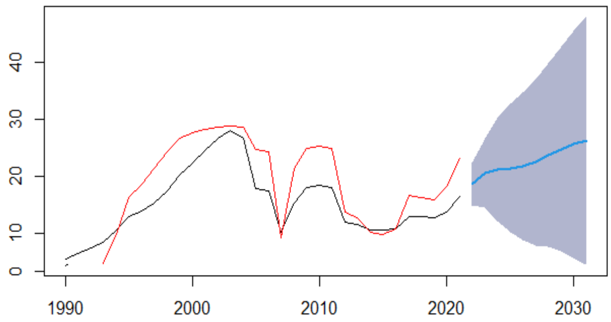

Fig. 4 Time series plot of the real, estimated, and predicted values of unemployment rates in Iraq

IV. CONCLUSION

The study of the characteristics of the time series of unemployment rates in Iraq for the period (1990-2021) showed that they are unstable, non-linear, and heterogeneous, and thus are suitable for use in applying non-linear time series models (Logistic, Gompertz, Richard, Chapman Richards). When comparing these models with each other, it was concluded that the best model for estimating unemployment rates is the Chapman-Richards model, as it yielded the lowest value for the predictive power measure (RMSE), indicating its prediction efficiency. When predicting unemployment rates in Iraq using the best model, we observe that unemployment rates have been consistently increasing from 2022 to 2031, despite the government's efforts to reduce unemployment in the country.

Therefore, we recommend utilizing this research to inform plans that address the unemployment problem in Iraq and increase job opportunities in both the public and private sectors, thereby accommodating the growing number of unemployed individuals. We recommend using the best model in this research to study other economic and social phenomena. We recommend increasing attention to collecting and documenting data accurately so that researchers can use it in future studies.

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