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E- Bayesian Estimation of System Reliability (Series, Parallel) and Failure Rate Functions with Kumaraswamy Distribution based on Type II Censoring Data

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***Abstract*— In this paper, the failure rate function and the shape parameter for the kumaraswamy distribution and reliability function of a system with a number (m) of independent compounds associated with a system (serial, parallel) were estimated, by relying on observational data of the second type, knowing that the survival time of the compounds are independent. Based on the findings the graphical predictor of the failure rate and parameter - and the reliability function of the serial and parallel system is smaller than the Standard Bayesian estimator (MLE) in simulation and real data. Thus, a decreasing in AMPE with an increase in the sample size n and an increase in the size of the failure sample r as the physical prediction capabilities have a high efficiency. The using of the Bayesian prediction method to estimate the reliability of different production systems for other failure distributions such as the Burr family distributions and various other failure distributions. Based on the output he results are reasonably consistent with simulation and real data. The E-Bayesian method was used for estimating with three primary distribution functions for the above parameters and comparing them with the standard Bayesian method with a squared loss function and the maximum likelihood method where simulation experiments were employed to compare the estimation results and the results showed the advantage of the E-Bayesian method in estimating through comparison statistics (MAPE).**

***Keywords*— bayesian estimation; system reliability; kumaraswamy distribution; censoring data; MAPE.**

I. INTRODUCTION

Life test experiments are interested in studying reliability data that are governed by some of the functions and probabilistic models, as there are many applications for life experience tests, industrial fields and other fields, and these processes have led to an increase in studies and researchers' interest in reliability, especially when the complexity of the systems is increasing and researchers are directed in their various studies To maintain devices and equipment and reduce the cost, and to demonstrate the efficiency of these devices in terms of work without breakdowns for the longest possible period and avoid sudden holidays This in turn leads to maintaining the optimum productivity of machines and equipment [1], [2].

As these studies and tests are conducted under many restrictions such as time and cost, failure times are usually not recorded for all elements that have been put under test. Such experiments lead to the emergence of data known as controlled data [3]. In many researches, different types of control data have been discussed, the most common The data of the control are of the first and second types, in this research the reliability function will be estimated based on the control data [4].

In 1980, Kumaraswamy proposed a two-parameter distribution [5].

$$f_T = \theta\,\gamma\,t^{\gamma-1}(1-t^{\gamma})^{(\theta-1)} \tag{1}$$

Whereas, γ and θ are parameters by form assuming that γ is known in this paper. As for the cumulative distribution function, it is as follows:

$$F_T(t;\theta) = 1-(1-t^{\gamma})^{\theta} \tag{2}$$

Then the reliability function and the failure rate distribution function will be sequentially as follows:

$$R(t) = (1-t^{\gamma})^{\theta} \tag{3}$$

$$h(t) = \frac{\gamma\theta t^{\gamma-1}}{1-t^{\gamma}} \tag{4}$$

II. MATERIALS AND METHOD

A. Systems Reliability

Reliability of systems is defined as a group of compounds and subsystems that are connected with each other so that the system performs the required functions and that this system works as long as its vehicles operate depending on the nature of the vehicles and subsystems and two types of common interconnection systems can be distinguished as follows [6]:

1) System Series: The serial connection of the system means that the system operates when all of its vehicles are operating, as the reliability of the system consisting of m of independent vehicles is as follows [7]:

$$R(t,s,m) = \prod_{i=1}^{m} R_i(t) \quad (5)$$

From equation (5) it can be noted that:

$$R(t,s,m) \leq min\{R_1(t).R_2(t) \ldots .R_m(t)\} \quad (6)$$

That is, the reliability of the serial system is less than the reliability of any vehicle in the system, and if the life time for each of the vehicles is not distributed by Kumaraswamy with the two parameters $\theta_i, i = 1,2,\ldots,m$ $\gamma_i, i = 1,2,\ldots,m$ ، Note that the life time is independent for each vehicle and the probability density function is as follows [8], [9]:

$$f(x,\theta_i,\gamma_i) = \theta_i\gamma_i(1-t^{\gamma_i})^{(\theta_i-1)} \quad (7)$$

Then the reliability of the distribution chain system is as follows

$$R_i(t) = (-t^{\gamma_i})^{\theta_i} \quad (8)$$

But if the components are identical, the system's reliability will be [5] :

$$R(t,s,m) = (1-t^{\gamma})^{m\theta} \quad (9)$$

2) Parallel System: The system is a factor if one of its vehicles remains active. At least, this type of connection is called parallel linking, where the system stops working when all its vehicles are stopped and the system reliability is as follows [10], [11]:

$$R(t,p,m) = 1 - [\left(1-R_1(t)\right)\left(1-R_2(t)\right) \quad (10)$$

$$R(t,p,m) = 1 - \prod_{i=1}^{m}\left(1-R_i(t)\right) \quad (11)$$

Where it is observed from equation 11 the following:

$$R(t,p,m) = max\{R_1(t).R_2(t).\ldots .R_m(t)\} \quad (12)$$

That is, the reliability of the parallel system is the greatest reliability of any of the system's components [12]. The reliability of the Kumaraswamy parallel system is as follows [13], [14]:

$$R(t,p,m) = \sum_{i=1}^{m}(1-t^{\gamma_i})^{\theta_i} \quad (13)$$

The Parallel system for Components Identical components is as follows:

$$R(t,p,m) = \sum_{j=1}^{m}(-1)^{j-1}\binom{m}{j}(1-t^{\gamma})^{j\theta} \quad (14)$$

The aim of this distribution is to estimate the shape parameter of Kumaraswamy distribution and the reliability, hazard rate functions with maximum likelihood, standard Bayesian, and E Bayesian Methods with three different kinds of priors under quadratic loss function.

B. Estimation Methods

For estimating the unknown parameters of the Kumaraswamy distribution, the reliability function of serial and parallel systems, and the failure rate function. The estimation methods are Maximum Likelihood, Standard Bayesian and E-Bayes Methods [15], [16].

C. Maximum Likelihood Method (MLE)

(MLE) is one of the important methods in the process of estimation, its properties and the most important of these properties is the property invariance[17]. In failure monitoring experiments, n of experimental units are placed under observation in a life-model test or product longevity at zero time (where time is a random variable that cannot be determined) and by specifying r of observations where r < n where data consists of observations $t_1,t_2,\ldots\ldots t_r$ that represent ages Test units This means that there is no information on survival units (n-r) except for those whose useful life is greater than t_r The test stops and the experiment ends when unit r fails [18]. Suppose that the first r of the failure times represent a sample that was previously determined with the size of n and placed under observation t is a random variable that distributes Kumaraswamy distribution with the parameters (θ, γ), but (MLE) is as follows :

$$L(\theta,\underline{t}) = \prod_{i=1}^{r} f(\theta,\gamma,t_i)[1-F(t_r)]^{n-r} \quad (15)$$

Substituting equation (1) and equation (2) in equation (15) we obtain the following:

$$L(\theta,\underline{t}) = \frac{n!}{(n-r)!}\prod_{i=1}^{r}\gamma\theta\, t_i^{\gamma-1} \quad (16)$$

$$L(\theta,\underline{t}) = \frac{n!}{(n-r)!}\prod_{i=1}^{r}[\frac{t_i^{\gamma-1}}{1-t_i^{\gamma}}]e^{-\theta R} \quad (17)$$

A parameter (MLE) is obtained for parameter θ, which is denoted by the symbol θ_m after taking the natural logarithm of equation (17) and its derivation with respect to θ. The estimator is as follows:

$\hat{\theta}_{ml} = \frac{r}{D}$ Then, the estimator of the System Series reliability function estimator can be found in the MLE, which will symbolize

To obtain the MLE estimator for the system parallel function, which is symbolized follows:

$$\hat{R}_{ml(p)} = \sum_{j=1}^{m}(-1)^{j-1}\binom{m}{j} \quad (18)$$

As for the MLE of the Function Rate Failure, which is denoted by a me, it is obtained by compensating equation (18) in equation (4) as follows:

$$\widehat{H}_{ml} = \frac{\gamma \widehat{\theta}_{ml} t^{\gamma-1}}{1-t^{\gamma}} \quad (19)$$

D. Standard Bayesian Method

In this section we will address the standard BES method of estimation, which assumes that the prior information that is available about the parameter to be estimated can be formulated as a function called the prior probability density function and this function is only an expression about the prior knowledge of this parameter [19].

Based on a sample data type II Type Censored with a size r, it was determined from n of the units under the test experiment that are subject to the distribution kumarasumy with the two parameters γ, θ and assuming that the parameter γ is known and assuming that the initial distribution of the parameter θ is the complete Gamma distribution [13]:

$$y(\theta/a,b) = \frac{b^{a}}{\Gamma(a)} \theta^{a-1} e^{-b\alpha} \quad (20)$$

Referring to the possibility function of observations and prior information and using Bayes theorem, the posterior distribution defined parameter θ as follows [20] :

$a \sim \mathrm{uniform}(0,1)$, $b \sim \mathrm{uniform}(0,c)$

$$z(\theta,\underline{t}) = \frac{L(\theta,\underline{t})y(\theta/a,b)}{\int_0^{\infty} L(\theta,\underline{t})y(\theta/a,b)d\theta} \quad (21)$$

$\theta \sim \mathrm{Gamma}(a,b)$

Then θ is to Bayesian estimator

$$Risk(\hat{\theta}) = \int_{\alpha}^{1} L(\hat{\theta},\theta) z(\theta,\underline{t}) d\theta \quad (22)$$

Using a quadratic loss function, which is defined as follows:

$$L(\hat{\theta},\theta) = (\hat{\theta}-\theta)^2 \quad (23)$$

Prediction of the posterior distribution

E. Bayesian Estimation Method

In this section of the research, the parameter θ, as well as the system reliability on method will be estimated in a way of the predictive prediction of the estimation based on a square error function and three different primary distributions of the hyper parameters b, a, and according to the researcher Han [4] , That these parameters should be chosen to ensure that the initial distribution function of the parameter θ given in equation (22) is decreasing for the parameter θ and that the derivative of prior distribution with respect to parameter θ is:

$$\frac{dy(\theta,a,b)}{d\theta} = \frac{b^{a}}{\Gamma(a)} \theta^{a-2} e^{-b\theta}\left[(a-1)-b\theta\right] \quad (24)$$

Where $\frac{dy(\theta,a,b)}{d\theta} < 0$ It can be seen that the derivative (36) of the function (θ, a, b) decreases to θ when it is $0 < a < 1$ and $b > 0$ Assuming that the parameters b, a are independent, they have the following joint prior distribution:

$$\pi(a,b) = \pi_1(a)\pi_2(b) \quad (25)$$

Therefore, the E Bayesian estimator for the parameter θ can be calculated as follows:

$$\hat{\theta}_{EBi=} \iint (a,b)\pi_i(a,b)dadb \quad i = 1,2,3 \quad (26)$$

Whereas (π_i (a, b) represents the joint prior probability distributions of the hyper parameters so that $0 < a < 1$, $0 < b < c$ where c is the upper limit of the parameter b and is chosen so that it does not move far from the value of the parameter a for the purpose of Maintaining the immunity of the Bayesian estimator ˙

III. Result and Discussion

In this section, a simulation model will be created from five stages to estimate the parameter of the shape theta on the assumption of parameter gamma as well as estimating the reliability function of the system (serial, parallel) and the failure rate function for distribution Kumaraswamy and detailed stages are as follows:

A. First Stage

In this stage, samples under probation n and sizes of failure samples rare set as follows:

We generate the values of the hyperparameters b, a and according to the formulas in equations (39) (40) (41). Assuming that $c = 2$, $\gamma = 2$. Also generate a default value for the parameter, according to the Gamma distribution

B. Second Stage

Generating control samples of the second type for distribution Kumaraswamy (the inverse function method) according to the following function:

$$t_i = F^{-1}(U_i) = [1-(1-F)^{\frac{1}{\theta}}]^{\frac{1}{\gamma}} \quad (27)$$

C. Third stage

We estimate the parameter θ by the methods mentioned in the theoretical side, according to the formulas (10) (16) (26) (27) (28).

D. Fourth Stage\

Estimating the reliability of the serial system by the methods mentioned in the theoretical side according to formulas (11) (19) (31) (33) (32)

E. Fifth stage

Estimating the reliability of the parallel system by the methods mentioned in the theoretical side according to formulas (12) (20) (35) (37) (36)

F. Sixth stage

Estimating the failure rate function by the mentioned methods in the theoretical side according to formulas (19) (31) (47) (49) (51).

G. Seventh stage

It is the stage in which the parameter value θ and its capabilities are compared according to the methods used in the research.

$$\mathrm{MAPE}(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\theta-\hat{\theta}_i}{\theta}\right| \quad (28)$$

Each experiment was repeated 1000 times (N = 1000). The following tables represent simulation results

TABLE I

SHOWS THE VALUES OF THE ESTIMATED PARAMETER Θ BY THE METHODS USED WHEN C = 2, Γ = 2

n	$\hat{\theta}_{mle}$	$\hat{\theta}_{B}$	$\hat{\theta}_{EB1}$	$\hat{\theta}_{EB2}$	$\hat{\theta}_{EB3}$
25	0.6300	0.6237	0.6888	0.6585	0.8876
	0.7077	0.7552	0.7361	0.7379	0.7363
40	0.6935	0.6989	0.6990	0.6999	0.6989
	0.7535	0.7555	0.7567	0.7559	0.7659
70	0.7836	0.7649	0.7533	0.7532	0.7530
	0.7976	0.7988	0.7983	0.7980	0.7990
100	0.8987	0.8975	0.9366	0.9389	0.9368
	0.9292	0.9381	0.9476	0.9470	0.9479

TABLE II

SHOWS (MAPE) FOR THE ESTIMATED PARAMETER Θ BY THE METHODS USED WHEN C = 2, Γ = 2

n	$\hat{\theta}_{mle}$	$\hat{\theta}_{EB1}$	$\hat{\theta}_{EB2}$	$\hat{\theta}_{EB3}$
25	0.5060	0.4848	0.4840	0.4839
	0.49077	0.4961	0.4960	0.4955
40	0.5044	0.4830	0.4825	0.4820
	0.4494	0.4356	0.4349	0.4325
70	0.4690	0.4536	0.4529	0.4519
	0.4276	0.4253	0.4249	0.4240
100	0.4587	0.4212	0.4209	0.4209
	0.4162	0.4076	0.4070	0.4066

TABLE III

SHOWS THE CAPABILITIES OF THE SERIAL SYSTEM RELIABILITY FUNCTION BY THE METHODS USED WHEN C = 2, Γ = 2

n	$\hat{R}_{ml(s)}$	$\hat{R}_{B_s}$	$\hat{R}_{EB_{s1}}$	$\hat{R}_{EB_{s2}}$	$\hat{R}_{EB_{s3}}$
25	0.1993	0.1951	0.1891	0.1853	0.1803
	0.2013	0.2008	0.1937	0.1904	0.1880
40	0.2037	0.19526	0.1935	0.1902	0.1885
	0.3365	0.33454	0.3331	0.3329	0.3323
70	0.2922	0.21176	0.2094	0.2023	0.1995
	0.3079	0.2362	0.2152	0.2142	0.2155
100	0.3026	0.23655	0.2149	0.2125	0.1992
	0.3835	0.24552	0.2293	0.2226	0.2197

TABLE IV

SHOWS THE CAPABILITIES OF THE PARALLEL SYSTEM RELIABILITY FUNCTION BY THE METHODS USED WHEN C = 2, Γ = 2

n	r	$\hat{R}_{B_p}$	$\hat{R}_{EB_{p1}}$	$\hat{R}_{EB_{p2}}$	$\hat{R}_{EB_{p3}}$
25	15	0.4987	0.4222	0.4205	0.4186
	20	0.4767	0.4125	0.4118	0.4106
40	25	0.4980	0.4218	0.4201	0.4180
	35	0.4901	0.4121	0.4109	0.4100
70	45	0.4955	0.4211	0.4202	0.4177
	55	0.4562	0.4153	0.4149	0.4145
100	75	0.4854	0.4134	0.4129	0.4122
	90	0.4381	0.4076	0.4078	0.4079

TABLE V

SHOWS THE CAPABILITIES OF THE FAILURE RATE FUNCTION BY THE METHODS USED

n	$\hat{h}_{mle}$	$\hat{h}_{B}$	$\hat{h}_{EB1}$	$\hat{h}_{EB3}$	$\hat{h}_{EB2}$
25	0.7366	0.7261	0.7250	0.7243	0.7244
	0.7034	0.7003	0.6351	0.6333	0.6349
40	0.7363	0.7258	0.7243	0.7238	0.7240
	0.7228	0.7251	0.7197	0.7150	0.7169
70	0.7186	0.7156	0. 7143	0. 7132	0. 7139
	0.7066	0.7052	0.7023	0.7010	0.7019
100	0.6887	0.6775	0.6666	0.6650	0.6657
	0.6792	0.6781	0.6776	0.6763	0.6770

A. Experiment procedure

In this aspect of the research, the application of all the estimation methods used in the experimental side has been carried out based on real data collected from the Central Organization for Standardization and Quality Control / Textile Industries Division, as these data that have been approved in this aspect represent towels and are from textile products .

1) Collecting and testing data

Here data was collected directly by taking a type of fabric, which is represented by towels, and examining it to find out the cut-off time for each unit measured per second using the force of fabric cutting device, as the test was a measure of the length of the strip of the towel, which is 20cm and a width of 5cm where the number of views selected was 25 views That is, the size of the sample (n = 25) recorded in the table below:

TABLE VI

SHOWS THE REAL DATA OF THE TOWELS, WHICH REPRESENTS THE CUT-OFF TIME FOR EACH UNIT MEASURED PER SECOND

S.	REAL DATA	S.	REAL DATA
1	12.43	14	12.48
2	12.39	15	13.13
3	13.52	16	12.91
4	13.62	17	13.32
5	12.73	18	12.95
6	13.91	19	13.17
7	13.21	20	12.81
8	13.29	21	12.65
9	13.83	22	14.15
10	14.12	23	14.21
11	14.31	24	13.81
12	13.43	25	13.66
13	12.18		

The data collected above are observational data of the second type and for the purpose of knowing whether the withdrawn data follow the kumaraswamy distribution according to the following hypothesis: The data are distributed kumaraswamy distribution: H_0 v.s. Data not distributed kumaraswamy distribution: H_1. Good conformity test Smirnov-Kolmogorov (S-K) for real data was performed using fit easy program and table VII below.

TABLE VII

SHOWS THE RESULTS OF THE KUMARASWAMY DISTRIBUTION OF THE REAL DATA

Kumaraswamy Distribution [#31]					
Smirnov-Kolmogorov test					
Sample Size Statistic P-Value Rank	25 0.1306 0.73944 46				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2079	0.23768	0.26404	0.29516	0.31657
Reject ?	No	No	No	No	No

Through the table above, we find that the values of the test-strength index (value-P) are greater than the levels of significance, as well as the value of the statistic in favor of the null hypothesis, and this indicates that the null hypothesis that the data is distributed a kumaraswamy distribution, and therefore the cut-off time data for the towels is distributed kumaraswamy distribution.

2) Data Analysis

After knowing the real data distribution represented by the kumaraswamy distribution, the estimation methods represented by the (MLE) method and the Standard Bayesian method were used based on a quadratic loss function and the Bayesian prediction method with three distributive functions for the meta-parameters for estimating the shape parameter for the distribution and the reliability function in the serial and parallel systems and the failure rate function, as shown in the table (8) Below, the Bayesian predictive method with its three functions is preferable to the rest of the methods, and this is equivalent to the experimental aspect.

TABLE VIII

SHOWS THE CAPABILITIES OF THE REAL DATA

Estimation methods / Functions and parameter	Standard Bayesian method	Bayesian prediction method for the primary function 1	Bayesian prediction method for the primary function 2	Bayesian prediction method for the primary distribution function 3
Shape parameter	$\hat{\theta}_{EB2}$ 0.1742	$\hat{\theta}_{EB1}$ 0.1749	$\hat{\theta}_{B}$ 0.1755	$\hat{\theta}_{mle}$ 0.1768
Functionality of the serial system	$\hat{R}_{EB_{s2}}$ 0.1722	$\hat{R}_{EB_{s1}}$ 0.1734	$\hat{R}_{B_s}$ 0.1777	$\hat{R}_{ml(s)}$ 0.1785
Functionality of parallel system	$\hat{R}_{EB_{p2}}$ 0.5051	$\hat{R}_{EB_{p1}}$ 0.5055	$\hat{R}_{B_p}$ 0.5074	$\hat{R}_{ml(p)}$ 0.5093
Failure rate function	$\hat{h}_{EB3}$ 0.5519	$\hat{h}_{EB1}$ 0.5521	$\hat{h}_{B}$ 0.5560	$\hat{h}_{mle}$ 0.5613

IV. CONCLUSION

In general, we find that the graphical predictor of the failure rate and parameter - and the reliability function of the serial and parallel system is smaller than the Standard Bayesian estimator (MLE) in simulation and real data. We notice from all tables that the E-Bayesian Estimation for a parameter in its three states tends to be more efficient than the Bayesian estimator & (MLE) for them because it has a smaller AMPE

We note a decrease in AMPE with an increase in the sample size n and an increase in the size of the failure sample r as the physical prediction capabilities have a high efficiency. We note that the results are reasonably consistent with simulation and real data, We recommend using the Bayesian prediction method to estimate the reliability of the serial and parallel system in applied studies for their efficiency in estimating. We recommend using the Bayesian prediction method to estimate the reliability of different production systems for other failure distributions such as the Burr family distributions and various other failure distributions.

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