

# Regulation of Nonlinear Chemical Processes with Variable Dead Time: a Generalized Proportional Integral Controller Proposal

William Chacón<sup>a,\*</sup>, Jefferson Vallejo<sup>a</sup>, Marco Herrera<sup>a</sup>, Oscar Camacho<sup>a</sup>

<sup>a</sup> Escuela Politécnica Nacional, Departamento de Automatización y Control Industrial. Quito, Ecuador

Corresponding author: \*william.chacon@epn.edu.ec

**Abstract**— This work shows the development and application of a Generalized Proportional Integral (GPI) Controller based on a First Order Plus Dead Time (FOPDT) approximation for an industrial chemical process. GPI control is a relatively recent advancement in automatic control. Nowadays, several enhancements with the GPI technique come from integral reconstructions of the system states. Chemical engineering processes present numerous challenging control problems, including nonlinear dynamic behavior. GPI can become a new option to consider in industrial applications since, along with the arrival of Industry 4.0, there are many improvements in computers and automation architecture to implement controller algorithms. The new controller's functionality shows significant accessibility of mathematical and logical potential. A comparison between the GPI and a PID controller is made under the same conditions to evaluate their performance. After carrying out some tests, the GPI shows better performance and a smoother controller action when applied to the mixing tank with a variable delay than the PID. Performance indexes as Integral Square Error (ISE) for evaluating the Output Variable and Control Effort Total Variation (TVu) for evaluating the control action are used to measure performance. Finally, designing an appropriate controller like the GPI that recognizes and incorporates nonlinearities is required for chemical processes. Simulations were developed using Simulink-MATLAB.

**Keywords**—Generalized Proportional Integral Control; chemical processes; First Order Plus Dead Time (FOPDT) approximation.

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## I. INTRODUCTION

Generalized Integral Proportional Control (GPI) control, or integral reconstructions-based control, is a recent development in the field of automatic control. Today, there are several developments with the GPI technique, which come from integral reconstructions of the system states. This GPI Control System can reject many types of structured disturbances. Within applications of this technique are, for example, static converters in power electronic systems with high-performance results. However, there is a weak or even non-existent development in chemical processes [1].

Chemical processes make chemical products from different inputs supported by equipment types by automatic or human controls. These industries work with different chemicals and petrochemicals and some materials like paper, minerals, steel, food, and even power generation systems. These applications share typical dynamics talking about continuous variables and depend on similar measurements, e.g., pressure, flow, level, and temperature [2]. Pumps and valves are commonly used as

conventional actuators. Chemical manufacturing processes pose several challenging control problems involving nonlinear dynamic behavior [3].

The present automation and control systems in industry and many process control units are founded principally on advances designed forty years ago, which did not change fundamentally in the last years. With the beginning of Industry 4.0, there are new challenges and new problems to solve, and many changes to automation architecture are still far, but on the way to be done. The new controller's functionality shows a large availability of mathematical and logical potential, e.g., auto-tuning potential and adaptive Proportional-Integral-Differential (PID) laws [4]. The PID controllers have been the most popular for process control in industries due to their remarkable simple structure, easy implementation, and standard tuning process [5]. Studies in the last years indicated that almost 90% of all processes in industries that involve controllers show a PID structure [5], [6].

On the other hand, the PID controllers' accomplishment is bounded in complex systems [2], [3]. One of the most critical process control problems is time delays [7]. It could cause some low-performance errors, non-convenient controller complexity, and systems instability, also if the systems contain varying time delays, produced as mixing consequences, wireless data communication protocols, or measurement lines. Sometimes, it could lead to poor stability and performance, and classical control cannot control the process [7].

Therefore, designing an appropriate controller that recognizes and incorporates the extent of nonlinearities is desirable for chemical processes [8], which is the primary purpose of this work. The main difference between GPI control and state feedback control lies in the absence of asymptotic observers [9], that is why GPI Controller proposes a structural state reconstruction [10]. Its main developments are the finite-dimensional linear systems with some extensions to nonlinear systems and linear delay differential systems [11]. The main difference between GPI control and state feedback control lies in the absence of asymptotic observers [9], that is why GPI Controller proposes a structural state reconstruction [10]. Its main developments are the finite-dimensional linear systems with some extensions to nonlinear systems and linear delay differential systems [11].

This paper describes a GPI controller's synthesis from a First Order Plus Dead Time (FOPDT) model and its application for chemical processes. For an appropriate result validation, a comparison between the proposed GPI-type controller and a PID is performed. Also, ISE (Integral Square Error) and TVu (Total variations of control efforts) [12] indexes evaluate the controllers' output performance and the smoothness of the controllers' actions, respectively.

After evaluating GPI control, it became a great option to overcome some of the limitations present in a PID controller process, such as low response accuracy against perturbations or reference changes in highly nonlinear systems and instability against modeling problems present in systems with variable delays. The paper is organized as follows. Section II addresses the material and method, including basic concepts and the development of the Generalized Proportional Integral (GPI) Controller and its control law. Then, Section III shows the result and discussion based on a case study. Lastly, Section IV shows the conclusions.

## II. MATERIAL AND METHOD

### A. First Order Plus Dead Time (FOPDT) Model

The proposed GPI control is designed from the FOPDT model. The goal of using this model is to simplify the GPI control design so that it can be implemented in a relatively more straightforward way. The FOPDT model is:

$$\frac{Y(s)}{U(s)} = \frac{K_p e^{-t_0 s}}{\tau s + 1} \quad (1)$$

Where:

- $U(s)$ : Input of the system
- $Y(s)$ : Output of the system
- $K_p$ : Gain of the system.

- $\tau$ : Constant time
- $t_0$ : Delay time

The parameters  $K_p$ ,  $\tau$  and  $t_0$  are obtained from the reaction curve for a 10% step input (see Fig. 1). The amplitudes  $\Delta u$  and  $\Delta y$  are used to calculate  $K_p$ , and the times at 28 % and 63.2% of the final value are used to calculate  $\tau$  and  $t_0$  [12].

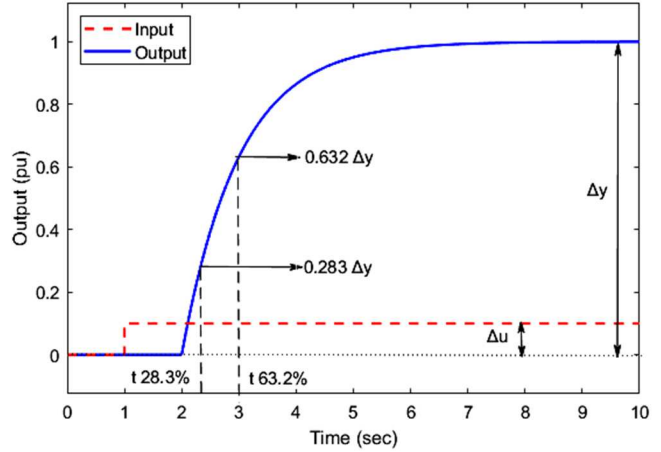


Fig. 1 Reaction curve

These times are chosen where the transient response presents rapid changes; thus, the model parameters are set precisely despite the measurement noise [12]. The expressions to calculate  $K_p$ ,  $\tau$  and  $t_0$  are shown below:

$$K_p = \frac{\Delta y}{\Delta u} \quad (2)$$

$$\tau = 1.5(t_{63\%} - t_{28\%}) \quad (3)$$

$$t_0 = t_{63\%} - \tau \quad (4)$$

### B. Taylor Approximation for Dead Time [13]

By using Equation (1), the Taylor approximation is performed, the numerator of the FOPDT is changed as follows:

$$e^{-t_0 s} = \frac{1}{t_0 s + 1} \quad (5)$$

$$G(s) = \frac{K_p}{(\tau s + 1)(t_0 s + 1)} \quad (6)$$

The expression (6) can be rearranged into

$$G(s) = \frac{K_p}{\tau t_0 \left( s^2 + \frac{\tau + t_0}{\tau t_0} s + \frac{1}{\tau t_0} \right)} \quad (7)$$

New variables A and B are included to simplify the expression,

$$A = \frac{\tau + t_0}{\tau t_0} \quad (8)$$

$$B = \frac{1}{\tau t_0} \quad (9)$$

$$G(s) = \frac{K_p B}{s^2 + A s + B} \quad (10)$$

Furthermore, a generalized model (11) is used as a function of the damping factor  $\varepsilon$  and the natural frequency  $\omega_n$

$$\frac{y(s)}{u(s)} = \frac{K\omega_n^2}{s^2 + 2\varepsilon\omega_n s + \omega_n^2} \quad (11)$$

Where  $K_p B = K$ . Expressions (10) and (11) are equalized to obtain

$$A = 2\varepsilon\omega_n = \frac{\tau + t_0}{\tau t_0} \quad (12)$$

$$B = \omega_n^2 = \frac{1}{\tau t_0} \quad (13)$$

Finally,  $\varepsilon$  and  $\omega_n$  are

$$\omega_n = \sqrt{\frac{1}{\tau t_0}} \quad (14)$$

$$\varepsilon = \frac{\tau + t_0}{2\tau t_0 \omega_n} \quad (15)$$

### C. Design of the GPI Controller from a FOPDT

For a simple nth order integration system of the form,

$$y^{(n)} = u \quad (16)$$

The control law is defined as [9]

$$u = u^* - \left[ \frac{k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + \dots + k_0}{s^{n-1} + k_{2n-2}s^{n-2} + \dots + k_n} \right] (y - y^*) \quad (17)$$

Its characteristic closed-loop polynomial is

$$p(s) = s^{2n-1} + k_{2n-2}s^{2n-2} + \dots + k_1s + k_0 \quad (18)$$

If a new integrator is added to obtain a robust system, the control law takes the following form [14].

$$u = u^* - \left[ \frac{k_n s^n + k_{n-1}s^{n-1} + \dots + k_1s + k_0}{s(s^{n-1} + k_{2n-1}s^{n-2} + \dots + k_{n+1})} \right] (y - y^*) \quad (19)$$

Whose characteristic polynomial is:

$$p(s) = s^{2n} + k_{2n-1}s^{2n-1} + \dots + k_1s + k_0 \quad (20)$$

The generalized control laws (17), (19), and the expression (10) are used to define the order of the controller. Since the system is second-order,  $n=2$ , the following GPI control law, and Robust GPI control law are respectively

$$u = \frac{1}{K} \left\{ u^* - \left[ \frac{k_1s + k_0}{s + k_2} \right] (y - y^*) \right\} \quad (21)$$

$$u = \frac{1}{K} \left\{ u^* - \left[ \frac{k_2s^2 + k_1s + k_0}{s(s + k_3)} \right] (y - y^*) \right\} \quad (22)$$

Where  $u^*$  is the initial condition of the control action,  $y^*$  is setpoint, and  $y$  is the system response.

The coefficients  $k_3, k_2, k_1$  and  $k_0$  are calculated based on the Hurwitz polynomial  $ph(s)$ . This includes a small parameter  $\varepsilon$  for the stability analysis of the system's observation error [14],

$$ph(s) = s^2 + \left( \frac{2\varepsilon\omega_n}{\varepsilon} \right) s + \frac{\omega_n^2}{\varepsilon^2} \quad (23)$$

In each case,  $ph(s)$  it must have the same order as the characteristic polynomial  $p(s)$ ; thus, the parameters  $k_3, k_2, k_1$  and  $k_0$  can be found by equating coefficients.

Noteworthy, the values of  $\varepsilon$  and  $\omega_n$  from (14) and (15) will be necessary for this section since all the other variables are dependent on them.

Table 1 shows the parameters for the GPI controller and the Robust one.

TABLE I  
GPI AND ROBUST GPI CONTROL PARAMETERS

GPI Controller		Robust GPI Controller	
Parameter	Formula	Parameter	Formula
$k_2$	$\frac{2\varepsilon\omega_n + 1}{\varepsilon}$	$k_3$	$\frac{4\varepsilon\omega_n}{\varepsilon}$
$k_1$	$\frac{2\varepsilon\omega_n + \omega_n^2}{\varepsilon^2}$	$k_2$	$\frac{4\varepsilon^2\omega_n^2}{\varepsilon^2} + \frac{2\omega_n^2}{\varepsilon^2}$
$k_0$	$\frac{\omega_n^2}{\varepsilon^3}$	$k_1$	$\frac{4\varepsilon\omega_n}{\varepsilon^3}$
		$k_0$	$\frac{\omega_n^4}{\varepsilon^4}$

The value  $\varepsilon$  is established based on the ISE and TVu performance indices. The value of  $\varepsilon$  is small and lies within  $0 < \varepsilon \leq 1$ . When the value  $\varepsilon$  approaches 1, the controller response is smooth but with longer settling times. The system response is fast in the opposite case when it tends to zero but with aggressive control actions [14].

Control schemes can be implemented within the simulation package of Simulink-MATLAB (see Fig. 2 and Fig. 3).

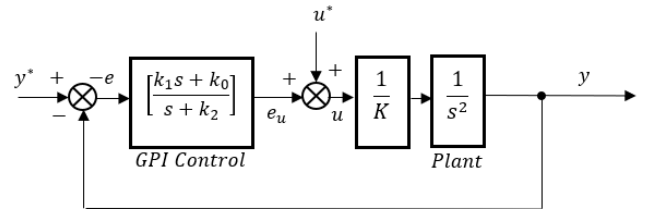


Fig. 2 GPI control scheme

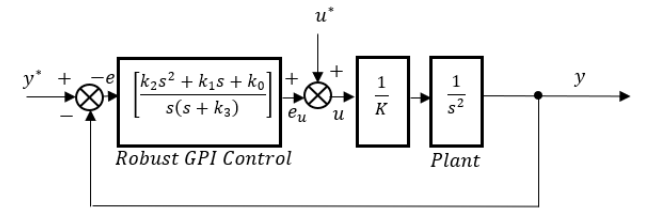


Fig. 3 Robust GPI control scheme

Here we have presented both the GPI and the Robust GPI controllers; however, only the Robust one will be used in the next cases. For more information about the development and Basis of GPI controllers and schemes, see [14].

## III. RESULTS AND DISCUSSION

### A. The Process with Variable Delay

The controller proposed is tested in a mixing tank, which enters two different water flows, one hot and one cold,

representing the process's inputs (see Fig. 4). A temperature transmitter measures the temperature in the output at 125 ft downstream [15].

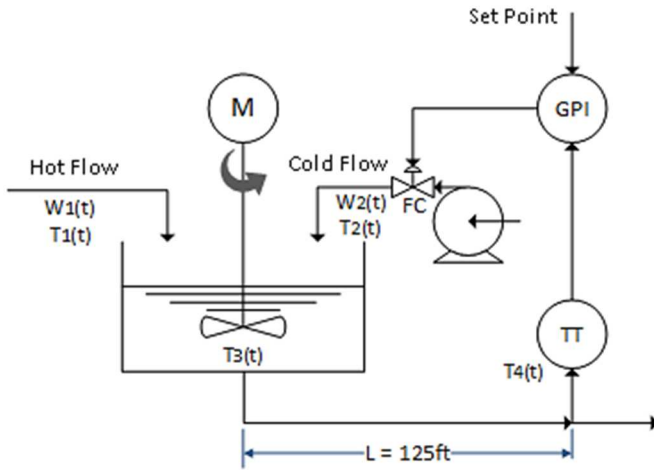


Fig. 4 Mixing tank

The next considerations should be taken:

- The tank is entirely isolated as well as the pipe.
- Inside the tank, the product is homogeneously mixed.
- The volume is constant in the tank.
- The operating range of the transmitter is in the range of 100 °F to 200 °F.
- The equations that describe the process are as follows:

$$VpCv_3 \frac{dT_3(t)}{dt} = W_1(t)Cp_1(t)T_1(t) + W_2(t)Cp_2(t)T_2(t) - (W_1(t) + W_2(t))Cp_3(t)T_3(t) \quad (24)$$

$$T_4(t) = T_3(t - t_0) \quad (25)$$

$$t_0 = \frac{LA\rho}{W_1(t) + W_2(t)} \quad (26)$$

$$ph(s) = s^2 + \left(\frac{2\varepsilon\omega_n}{\varepsilon}\right)s + \frac{\omega_n^2}{\varepsilon^2} \quad (27)$$

$$\frac{dT_O(t)}{dt} = \frac{1}{\tau_T} \left[ \frac{T_4(t) - 100}{100} - T_O(t) \right] \quad (28)$$

$$\frac{dV_p(t)}{dt} = \frac{1}{\tau_{V_p}} [m(t) - V_p(t)] \quad (29)$$

$$W_2 = \frac{500}{60} C_{VL} V_p(t) \sqrt{G_f \Delta P_v} \quad (30)$$

Where,  $W_1(t)$  and  $W_2(t)$  are the hot and cold input flow respectively,  $Cp$  represents the caloric capacity of a liquid at constant pressure in Btu/lb – °F,  $Cv$  is the caloric capacity of the liquid at constant volume in Btu/lb-°F,  $T_1(t)$  is the temperature of the hot fluid in °F,  $T_2(t)$  is the temperature of the cold fluid in °F,  $T_3(t)$  is the temperature of the fluid in the tank in °F,  $T_4(t)$  is the same temperature  $T_3(t)$  only that delayed by the time  $t_0$ ,  $t_0$  is the delay time expressed in min,  $\rho$  is the density of the substance already mixed in the tank in  $lbm/ft^3$ ,  $V$  represents the volume of the tank in  $ft^3$ ,  $T_O(t)$  is the transmitter output signal whose value is in a range

between 0 and 1,  $V_p(t)$  indicates the position valve range 0 (closed) to 1 (open),  $m(t)$  is a value between 0 to 1 that indicates a fraction of the controller output,  $C_{VL}$  is the valve flow coefficient expressed in  $gpm/psi^{1/2}$ ,  $G_f$  is the specific gravity,  $\Delta P_v$  represents the pressure drop in the valve in  $psi$ ,  $\tau_T$  is the time constant of the temperature sensor measured in min,  $\tau_{V_p}$  is the actuator time constant also measured in min,  $A$  is the cross-section of the pipe in  $ft^2$ . Furthermore, finally,  $L$  is the length of the pipe.

The operation conditions can be found in Table 2.

TABLE II  
STEADY/STATE VALUES

Variable	Value	Units	Variable	Value	Units
$W_1$	250	lb/min	$V$	15	$ft^3$
$W_2$	191.17	lb/min	$T_O$	0.5	-
$Cp_1$	0.8	Btu/lb – °F	$V_p$	0.478	-
$Cp_2$	1	Btu/lb – °F	$\bar{m}$	0.478CO	-
SP	150	°F	$C_{VL}$	12	Gpm/psi
$T_1$	250	°F	$\Delta P_v$	16	psi
$T_2$	50	°F	$\tau_T$	0.5	min
$T_3$	150	°F	$\tau_{V_p}$	0.4	min
$\rho$	62.4	lb/ft <sup>3</sup>	$A$	0.2006	ft <sup>2</sup>
$L$	125	ft			

#### A. First Order Approximation

The reaction curve is obtained following the procedure presented in [16]. It is recommended to perform the same procedure for a -10% change to the input signal. If the values of the obtained variables with this process coincide remarkably with the ones obtained in the positive change previously made, then the variables obtained in this last step are discarded; if not, an average of all these values are carried out among all the values obtained.

Once the data has been obtained, we process it to find the nonlinear system's transfer function. Thus, Fig. 5. shows the reaction curve for a positive 10% change in the value of the input signal.

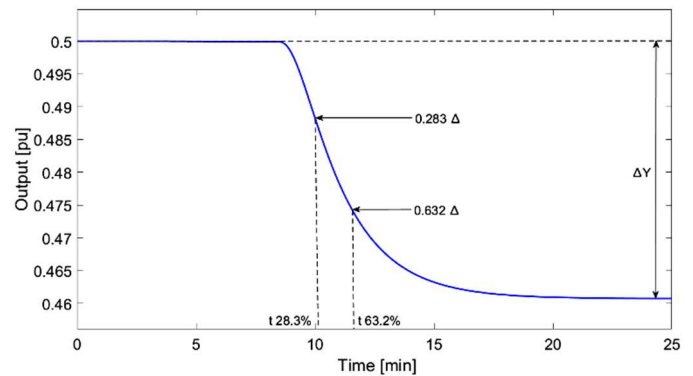


Fig. 5 Output curve for a positive 10% change in the input signal

Fig. 6 shows the reaction curve for a negative 10% change in the input signal.

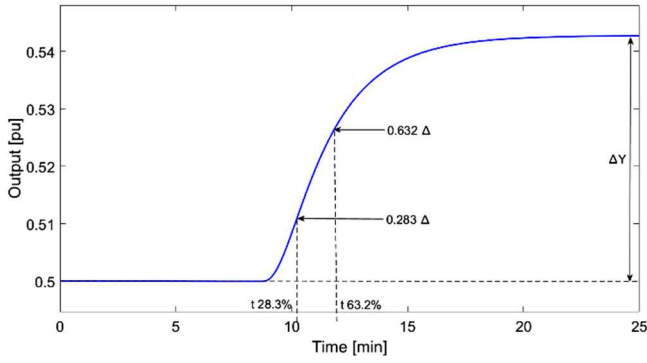


Fig. 6 Output curve for a negative 10% change in the input signal

Now, a transfer function is obtained:

$$G(s) = -\frac{0.8577e^{-4.36825s}}{2.30925s + 1}. \quad (31)$$

### B. Tuning Process

From the FOPDT model (8), the PID controller is tuned using the Dahlin adjustment formulas [12].

$$k_p = \frac{1}{2K} \left( \frac{t_0}{\tau} \right)^{-1} \quad (32)$$

$$T_i = \tau \quad (33)$$

$$T_d = \frac{t_0}{2}. \quad (34)$$

The PID parameters are the ones shown in Table 3.

TABLE III  
PID PARAMETERS

Parameter	Value
$k_p$	-0.31
$T_i$	2.31
$T_d$	2.18

For the GPI Controller, using the equations (14) to (15) and Table 1., the tuning parameters' values are shown in Table 4.

TABLE IV  
TUNING PARAMETERS FOR GPI CONTROLLER

Parameter	Value	Parameter	Value
$\omega_n$	0.31	$k_1$	1.05
$\varepsilon$	1.05	$k_0$	0.1572
$k_3$	2.6479	$\epsilon$	0.5
$k_2$	2.5459		

### C. Simulation and Analysis results

A disturbance in the flow of hot water is considered changing from the steady state of 250 [lb/min] to 200 [lb/min], 175 [lb/min], 150 [lb/min], and finally to 125 [lb/min] at times 10, 125, 250, and 425 [min], respectively (see Fig. 7.).

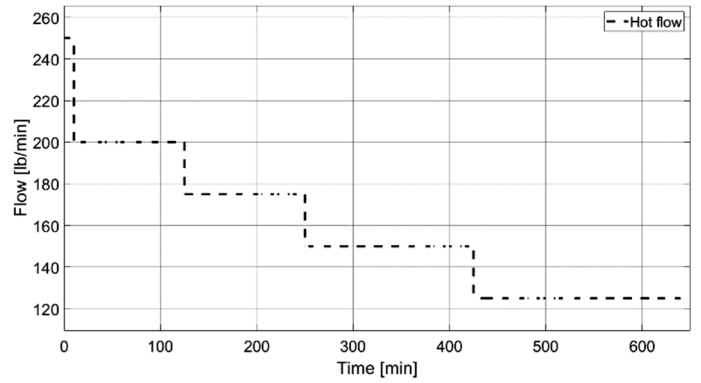


Fig. 7 Change in hot water flow

Fig. 8 shows the process's output against the disturbances produced in the hot water input flow. Compared with the PID Controller, when disturbances occur, the GPI controller compensates for the error better. However, this result shows that increasing the delay time results in a modeling error and deteriorates the controllers [16].

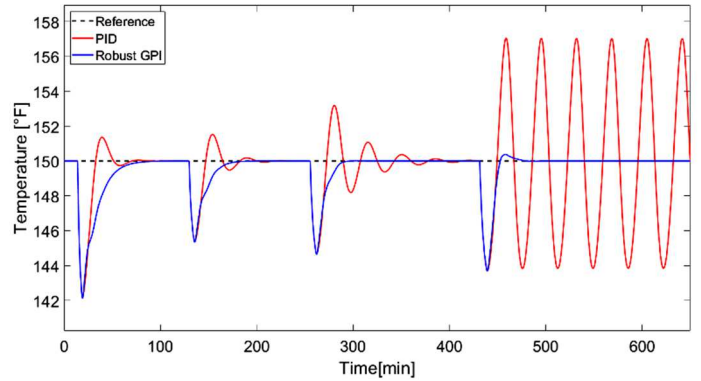


Fig. 8 Process output against disturbance

The GPI Controller reacts better than the PID even in the last disturbance at 425 [min]; in that time, the PID loses control of the system, becoming critically stable. Next, we see both controllers' control actions in Fig. 9, where the GPI Controller reacts smoothly than the PID.

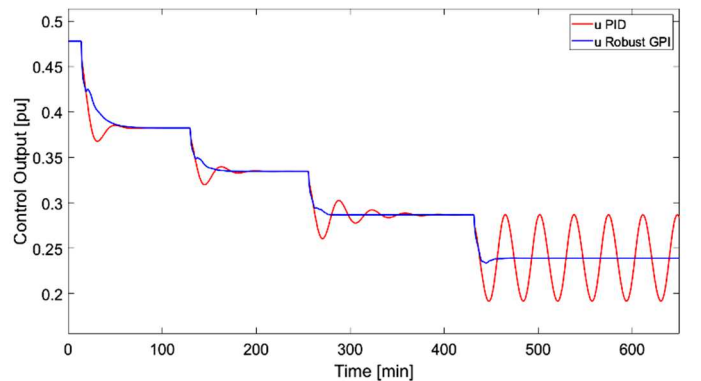


Fig. 9 Controller output in the mixing tank

Table 5 shows the ISE and TVu indexes values for GPI and PID Controllers.

TABLE V  
ISE COMPARISON

	ISE	TVu
<b>PID</b>	0.5757	$27.98 \times 10^{-3}$
<b>GPI</b>	0.1387	$8.67 \times 10^{-3}$

Fig. 10 shows the ISE comparison within PID and GPI Controllers.

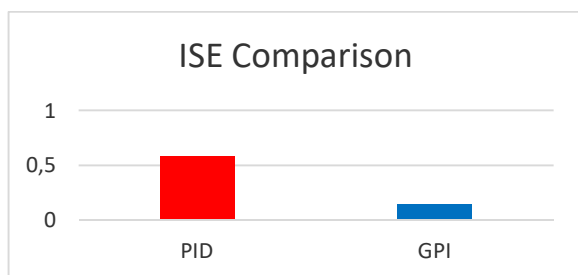


Fig. 10 ISE Index Comparison within PID and GPI Controllers.

Fig. 11 shows the TVu comparison between PID and GPI Controllers.

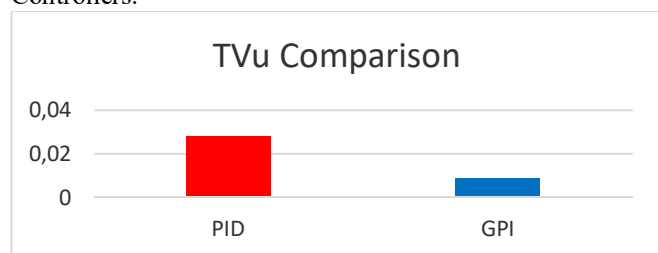


Fig. 11 TVu Index Comparison within PID and GPI Controllers.

The resulting graphs show that the best performance against disturbances is obtained for the GPI Controller since it presents better ISE and TVu performances. Additionally, faster response and smoother control action compared to the PID control.

#### IV. CONCLUSION

This paper presents the design of a GPI controller from a FOPDT model of a process. The controller resulted in a faster response with a smoother control action than PID, as demonstrated by the ISE and TVu performance indexes values in each case. Robust GPI control is a good option for this type of process. However, if better results are required, it is necessary to re-tune using the parameter  $\epsilon$  based on the ISE and TVu performance indexes.

Appropriate tuning of the controller makes the final element work better, reducing response times and prolonging its useful

life. The controller approach benefits from being flexible and straightforward, as it has been established for any process that a FOPDT model can characterize. These characteristics make their industrial application extremely feasible. Therefore, the controller law should be relatively simple to execute in any DCS [6].

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