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Forecasting Mortality Trend of Indonesian Old Aged Population with Bayesian Method

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***Abstract*— From around the nineteenth until the beginning of the twenty-first century, mortality rates show a declining trend. However, recent data on the United States population shows that the rate of decline started to slow down in the 2010s. Insurance companies need to be prepared in both ways: either mortality rates continue to decline, or there will be a turning point, and mortality rates start to increase. In this paper, we aim to get the whole picture of the mortality trend of Indonesian males, detect the possibility of a turning point in the mortality rates, and forecast mortality rates in the future. To reach this aim, we propose adjustments to the Makeham mortality model by including period and cohort information of the population via quadratic function. We also propose using the Bayesian method to estimate the parameters for the Indonesian old-aged males' population, where some adjustments were made in determining the priors, and the estimates were sampled from the posterior distribution using the Gibbs sampling algorithm. We found that our forecasting accuracy is satisfactory by considering the mean absolute percentage error values and coefficient of determination (R^2). We found that mortality rates are declining in the long term, but the probability of a turning point in the future is statistically significant. We identified two risks, longevity risk because of more centenarians in the future and mortality risk before their children complete compulsory education.**

***Keywords*— Longevity risk; Makeham model; mortality rates.**

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I. INTRODUCTION

Life insurance is an important asset for its protection benefits, especially for individuals who have debt, have or are planning on starting a family, or own a business. Even though modern insurance policies (for example, unit-linked insurance) are getting popular in Indonesian society, there is still a considerably large portion (around 40%) of insurance customers who choose the traditional one [1]. Traditional insurance policies assume that demographic uncertainty dominates the investment uncertainty [2].

Two important factors about demographic uncertainty in actuarial studies are mortality risk and longevity risk. Insurance companies need to set appropriate risk charges to keep being profitable. The mortality risk charge compensates the insurer for any losses as a result of the death of the policyholder [3]–[5] and affecting insurer's surplus [6]–[11]. While longevity risk exposes insurance companies to the chance that they have greater-than-anticipated cash flow needs on pension funds [12]–[15] and even those who were involved with longevity risk in Europe and North America started to increase their participation in the longevity risk transfer market rapidly [16].

Life expectancy has increased rapidly since the Age of Enlightenment, and global inequality regarding health conditions has decreased over the last decades [17]. Their estimates suggest that life expectancy was around 30 years before the 19th century; it has doubled to above 70 years since 1900. The trend of living ever longer and healthier lives seemed assured, but current mortality rates in the United States showed that the trend has slowed, starting from 2010 [18]. The decline in mortality rates has slowed, and it started to increase since 2015 in the United States. Several reasons for the recent trend change are slowly growing, stagnant and even declining incomes, drug overdoses, suicides, and alcohol-related liver mortality. Chandra and Abdullah [19] suggested the possibility that mortality rates for males in the 2015-2020 study period are not going to be lower than those in the 2005-2010 or 2010-2015 periods.

Inspired by United Nations [20] and [21], we aimed to forecast five-years abridged mortality rates for Indonesian old-aged males and females aged 30-95 years old from 2020-2025 period until 2045 2050 period. We hope that this

forecast is useful for stakeholders related to the Indonesian population, for example, managing insurance and the health system [22].

II. Materials and Method

This section describes the data we used in this study. We also explain ex-works related to our study objectives and what we learned to improve our formulation. After that, we discuss our modeling approach to be implemented and provide the results.

A. Data Description

We use data on Indonesian males and females' abridged life table for five years study period, starting from 1950 – 1955 to 2015 – 2020, which was published as a part of World Population Prospects 2019 [20], [21]. In this research, we considered values of ${}_5q_{30}$, ${}_5q_{35}$, ${}_5q_{40}$, ${}_5q_{45}$, ${}_5q_{50}$, ${}_5q_{55}$, ${}_5q_{60}$, ${}_5q_{65}$, ${}_5q_{70}$, ${}_5q_{75}$, ${}_5q_{80}$, ${}_5q_{85}$, ${}_5q_{90}$, and ${}_5q_{95}$ to represent the old, aged population. Therefore, the oldest population to contribute to the dataset was born in the 1855-1860 period, and the youngest population to contribute to the dataset was born in the 1985-1990 period.

To test the forecasting ability of our model, we divided our data into a training set and testing set for each sex. Training set consists of ${}_5q_x$ values with age denoted as $x \in [30, 80]$, contributed by populations that were born in 1960-1965 period or before, and their mortality event were studied in 2000-2005 period or before. The testing set consists of all values that are not part of the training set. Therefore, we have 118 values in the training set and 78 values in the testing set for each sex, giving us a rough ratio of 60:40.

B. Related Works and Study Objective

Hunt and Blake [23] reviewed some age/period/cohort (APC) models that were implemented to study the evolution and projection of mortality rates. While there are several age models in non-parametric and parametric approaches, to the best of our knowledge, studies on time models in parametric approaches with considerably simple fitting processes and interpretability are limited. As a result, the time representation must be forecasted further. For example, Safitri *et al.* [24] implemented a feedforward neural network to project future kt values in the Lee-Carter model. Also, Dong *et al*. [25] also implemented tensor decomposition.

As a result of further forecast of the time representation, it is hard to understand the general trend of mortality rates. Moreover, the calculation must be done in two steps: 1) estimate the values of deterministic parameters, and 2) project the values of stochastic parameters. A study by Qiao and Sherris [26] tried to solve the issue by implementing a linear function of time in the regression formula, each by stochastic and deterministic approaches, respectively. Although the approach is considerably simple and can predict the increase or decrease in mortality rates, it did not test the possibility of a turning point. Qiao and Sherris [26] also assumed that the force of mortalities is normally distributed. However, we argue that the assumption fails to fulfill the normality requirement since the force of mortalities must be positive valued for all ages. Hilton *et al.* [27] implemented the spline function of time, and it was more flexible than the linear function, but it lacks the ease of interpretation.

An important issue that we identified is that the results from previous studies were only in point estimates, which we argue lack the reliability of interpretation. Even if we have large samples, there is no reason we should expect a point estimate from a given sample to be exactly equal to the condition of the population. Therefore, it is preferable to determine an interval estimate of the parameters of interest. Bayesian method is considered due to the flexibility of this method that incorporates experts' judgment in addition to the data. Therefore, an optimal result could still be obtained even under the circumstances of low qualified data [28]–[31]. Chandra and Abdullah [19] produced annual mortality rates of the Indonesian population, for both males and females, in the form of interval estimates using the Bayesian method with Metropolis-Hastings algorithm and bootstrapping. However, this study was based on 2015-2020 data, a relatively short period, and not provide future projections.

General-purpose of this study is to obtain the whole picture of the mortality trend for Indonesian males. To reach the purpose, we have two specific objectives: to detect the possibility of a turning point in the mortality trend and forecast future mortality rates. To achieve the research aim and objectives, we propose the following framework. First, we will develop a method that can be used for forecasting (i.e., extrapolation). Second, a further model that could provide insights (i.e., allows for interpretation) on the mortality trend will be proposed. Third, we will address the trade-off between data size and model complexity to guarantee the validity of the results. Fourth, once the model parameter estimates are obtained, we will develop an approach to construct the confidence interval for each estimate. Finally, we apply the constructed approach to Indonesian population data, both males and females, to analyze the probability of the turning point in the mortality trend and to forecast future mortality rates.

In this study, we propose a simpler method than those previously discussed at the beginning of this section, in the sense that this method needs only one step of the calculation process. However, the proposed method will test the possibility of slowing mortality decline, is easier to interpret, and can provide interval estimates. Learning from Dong *et al.* [25], we decided to divide our data into a training set for model building and a testing set to assess the model's performance, thus confirming the robustness of the model to forecast new data.

C. Research Method

In this study, we assume the compliance of the Makeham mortality model and adjust that model for forecasting purposes, which will be elaborated more in the subsequent sections. We consider two independent models for each sex and one pooled model for both sexes. We aim to identify and assess whether the mortality trends differ according to gender by considering these three models. Moreover, we will estimate a 95% credible interval of the parameters and measure the accuracy of the fitted model on testing data. Finally, we provide a forecast of mortality rates for the next periods. If there are no trend differences, the reported metrics will apply for both sexes. A summary of the detailed process is displayed in Fig. 1.

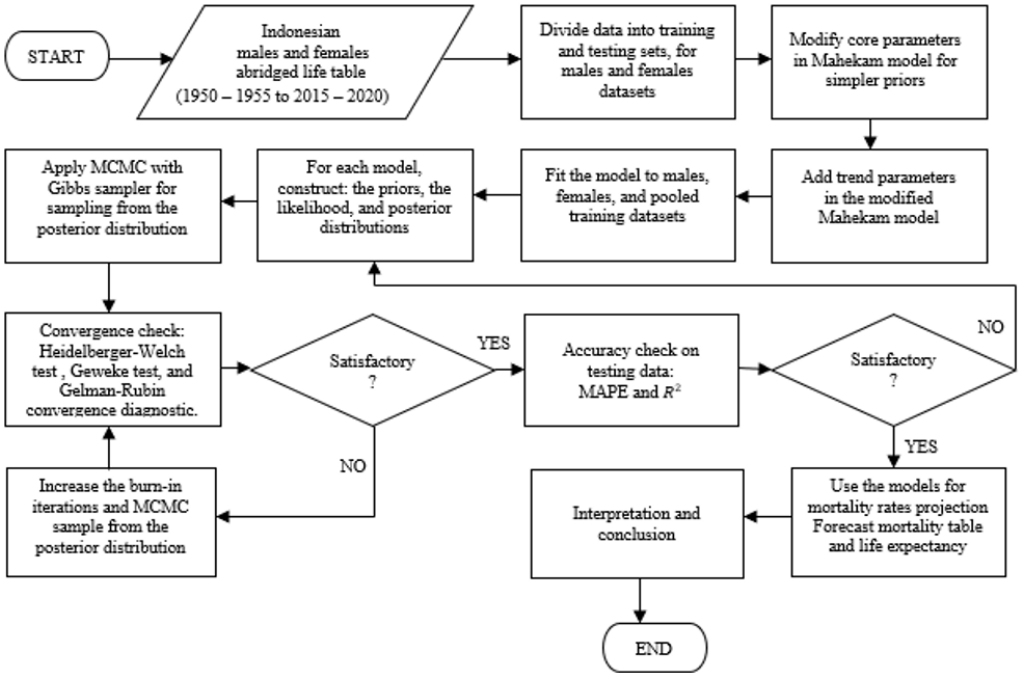

Fig. 1 Flowchart of the research method

1) Makeham model adjustment for forecasting purposes with separate sex: According to Dickson *et al.* [2], the Makeham model is defined by expressing the force of mortality at age x (μ_x) as written in (1).

$$\mu_x = A + BC^x, A > -B, B > 0, C \geq 1 \quad (1)$$

where A is an age-independent parameter, but B and C are age-dependent parameters. This specification indicates that the model fits well when mortality rates are increasing exponentially in terms of age.

According to Jordan [32], usually, the value of A is in the range of (0.001, 0.003), B is in the range of (10^{-6}, 10^{-3}), and C is in the range of (1.08, 1.12). Good fit usually occurs for the age range of 30 to 80 years old, and actuarial practitioners often consider this model for its extrapolation ability. Chandra and Abdullah [19] modified the model as served by (2) to simplify the construction of prior distributions and parameter estimation process in the Bayesian method.

$$\mu_x = (\alpha - B) + B(\varsigma + 1)^x, \alpha > 0, B > 0, \varsigma > 0 \quad (2)$$

Referring to the latter model specification, Chandra and Abdullah [19] expressed α as the sum of A and B, also ς is C subtracted by one. Referring to the range suggested by Jordan [32], the value of α is usually in the range of ($0.001 + 10^{-6}$, 0.004) and ς is in the range of (0.08, 0.12). It implies that α is currently influenced by both age-independent and age-dependent factors. We interpret that age-independent factors are not related to the individuals and cannot be explained well by their information. Yet, they are supposed to be related to overall environmental conditions. Therefore, we assign the time of the study period to represent the age-independent factors. In comparison, age-dependent factors are related to individuals and can be explained well by their information. Thus, we assign cohort information to represent the age-dependent factors.

Considering the year of birth, our data starts with the 1855-1860 period and ends in the 1985-1990 period. We converted these periods into numerical predictors, denoted as y, referring to the 1855-1860 period as the base. Therefore, y equals zero for the 1855-1860 period, one for the 1860-1865 period, and will continue as such. As for the study period, our data starts with the 1950-1955 period and ends with the 2015-2020 period. We converted these periods into numerical predictors, denoted as z, referring to the 1950-1955 period as the base. Therefore, z equals zero for the 1950-1955 period, one for the 1955-1960 period, and will continue as such.

Our interest is in understanding the overall trend of mortality rates and the existence of a turning point, both caused by age-independent factors (represented by time of study period) and age-dependent factors (represented by cohort information). We choose a quadratic form of mathematical expressions to investigate this occurrence and keep the model to be parsimonious. Since α, B, and ς should be positive for all cohorts and study periods, we seek a mathematical function that preserves this condition for all inputs. Therefore, we choose the exponential function, which also maintains the monotonicity property to guarantee the ease of interpretation. We express α, β, and ς as in (3), (4), and (5).

$$\alpha_{y,z} = \alpha_0 \exp(\alpha_1 y + \alpha_2 y^2 + \alpha_3 z + \alpha_4 z^2) \quad (3)$$

$$B_{y,z} = \beta_0 \exp(\beta_1 y + \beta_2 y^2) \quad (4)$$

$$\varsigma_{y,z} = \varsigma_0 \exp(\varsigma_1 y + \varsigma_2 y^2) \quad (5)$$

By substituting y and z with zero, α_0, β_0, and ς_0 could be interpreted as the value of α, B, and ς, respectively, for those who were born in 1855-1860 period and were studied in 1950-1955 period. The study period does not affect the mortality rates if the values of α_3 and α_4 equal zero. While cohort effect does not affect the mortality rates if and only if the values of α_1, α_2, β_1, β_2, ς_1, and ς_2 are all equal to zero. As a result, we

will have a model with eleven parameters (i.e. α_j, β_k, ς_l, j = 0,…,4; k = 0,…,2; l = 0,..,2) and three predictors (i.e. x, y, and z).

Our data do not have the values of μ_x; instead, we have data on ${}_5q_x$. Our model assumes that mortality rates are determined by age (x), index of five-year birth year period (y), an index of the five-year study period (z). Therefore, the final model that we propose to fit the data is adjusted as in (6).

$$\begin{aligned} {}_5q_{x,y,z} = 1 - exp\Big(-5\big(&\alpha_0 e^{\alpha_1 y+\alpha_2 y^2+\alpha_3 z+\alpha_4 z^2} \\ &- \beta_0 e^{\beta_1 y+\beta_2 y^2}\big)\Big) \\ \times \exp&\left(-\frac{\beta_0 e^{\beta_1 y+\beta_2 y^2}}{\log(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2}+1)}\right) \\ \times \exp&\left(\left(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2}+1\right)^x\left(\left(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2}+1\right)^5-1\right)\right) \end{aligned} \quad (6)$$

Once the model is specified, estimation of its parameters is required, as discussed in the subsequent section. The fitting process run in two phases, one for males and one for females. We assume that the parameters for males and females are independent, so we state this model as an independent sex model in the later sections.

2) Gibbs sampling to estimate the parameters of independent sex models: The equation of ${}_5q_{x,y,z}$ is considerably complicated to estimate the parameters. Since we have information on the domain, the suggested range, and previous research results of parameter values, implementing the Bayesian method could optimally utilize this information. We use the Gibbs sampler provided in WinBUGS [33] and R2WinBUGS [34] to simplify the estimation process.

Chandra and Abdullah [19] considered the studies of Australian, British, Indian, and Malaysian populations to construct their prior distributions. After fitting the Makeham model into data, specifically from the 2015-2020 study period, the 95% credible intervals for all parameters were narrower than the 95% confidence intervals of their prior distributions and the range proposed by Jordan [32]. Therefore, we decided to construct new distributions after considering 95% credible intervals estimated by Cox *et al.* [9] and the implied range by Jordan [32], starting from α_0, B_0, and ς_0. Selected prior distributions are two-parameters Weibull and lognormal with parameterization follows.

We still need to construct prior distributions for parameters representing the effects of cohort and study period on the trend of mortality rates. By looking at our study objectives and current visible decreasing trend of mortality rates, our worst scenario assumes that the initial values of α, B, and ς will fall at the 2.5-percentile of prior distributions (respectively for α_0, β_0, and ς_0) and final values of $\alpha_{32,19}$, $B_{32,19}$, and $\varsigma_{32,19}$ (for a population who is born in 2015-2020 period and will be studied in 2045-2050 period) will fall at the 97.5-percentile of prior distributions (respectively for α_0, β_0, and ς_0). We do not provide any prior tendency related to how cohort and study period affect mortality rates, positively or negatively. Therefore, our prior distributions are constructed based on a normal distribution with parameterization follows.

We follow the specification of the sampling model constructed by Chandra and Abdullah [19]. Our trust in the data is considerably low since we only have the mortality rates without underlying data to calculate them. The variance of the sampling model is to be maximized to represent our belief without causing the problem of underflow or overflow in the fitting process. Thus, by denoting ${}_5Q_{x,y,z}$ as a random variable representing the value of ${}_5q_{x,y,z}$, our sampling model for a particular mortality rate is defined as (7).

$$({}_5Q_{x,y,z}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\varsigma}) \sim \text{Beta}\left(1, \frac{1-{}_5q_{x,y,z}}{{}_5q_{x,y,z}}\right) \quad (7)$$

where $\boldsymbol{\alpha} = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4\}, \boldsymbol{\beta} = \{\beta_0, \beta_1, \beta_2\}$, and $\boldsymbol{\varsigma} = \{\varsigma_0, \varsigma_1, \varsigma_2\}$.

By assuming conditional independence on the data, we could construct our full sampling model as a multiplication of sampling model for each mortality rate. The posterior joint density of the parameters is proportional to the product of the density function of the parameters and the full sampling model. Denoting our training data as $\{q_1, q_2, \ldots, q_{118}\}$, our posterior joint density could be expressed by (8).

$$\begin{aligned} &\text{Posterior} \propto \alpha_0^{-0.675124}\beta_0^{-1}\varsigma_0^{-1} \times \\ &e^{-\frac{1}{2}\left\{\sum_{i=1}^{4}\left(\frac{\alpha_i}{5.45\times10^{-3}}\right)^2+\sum_{i=1}^{2}\left(\frac{\beta_i}{3.34\times10^{-3}}\right)^2+\sum_{i=1}^{4}\left(\frac{\varsigma_i}{1.96\times10^{-4}}\right)^2\right\}}\times \\ &e^{-\frac{1}{2}\left\{\left(\frac{log\,\beta_0+13.997}{7.271}\right)^2+\left(\frac{log\,\varsigma_0+2.536}{0.427}\right)^2\right\}}\times \\ &e^{-22.178\alpha_0^{0.325}}\prod_{i=1}^{118}\left(\frac{\Gamma\left(\frac{1}{q_i}\right)}{\Gamma\left(\frac{1-q_i}{q_i}\right)}(q_i)^{\frac{1}{q_i}-1}(1-q_i)^{\frac{1}{q_i}-2}\right) \end{aligned} \quad (8)$$

We fitted all data together because there is a possible correlation between them. Therefore, we suggest that it is more plausible to estimate the initial values and trend parameters together in one model than fitting independent Makeham models into each part of data and forecast every parameter independently by ARIMA models later.

3) Makeham model adjustment for forecasting purposes with pooled sex models: Independent models for each sex produced better accuracy than pooled sexes model, but they also have disadvantages. First, they take a longer duration to have the fitting process. Second, some parties may consider that overall degrees of freedom is smaller than pooling both sexes into a model. Therefore, we construct the pooled model by modifying (3), (4), and (5). Assuming that the estimate of trend parameters are equal for both males and females, our expressions for the value of α, β, and ς in the pooled model are expressed as (9), (10), and (11) with adding a sex indicator denoted by s, with the value of 1 for males and 0 for females.

$$\alpha_{y,z,w} = \alpha_0\, exp(\alpha_1 y + \alpha_2 y^2 + \alpha_3 z + \alpha_4 z^2 + \alpha_5 s) \quad (9)$$

$$B_{y,z,w} = \beta_0\, exp(\beta_1 y + \beta_2 y^2 + \beta_3 s) \quad (10)$$

$$\varsigma_{y,z,w} = \varsigma_0\, exp(\varsigma_1 y + \varsigma_2 y^2 + \varsigma_3 s) \quad (11)$$

The values of α_5, β_3, and ς_3 represent the naturally-log ratio of α, β, and ς for males to females, respectively. We expect that the differences are small, so we put normal distribution with zero mean as the prior distribution of α_5, β_3, and ς_3 (later, we mention them as ratio parameters). The standard deviations are determined by looking at the naturally-log form of the ratio between 97.5-percentile to 2.5-percentile for every prior distribution of α_0, β_0, and ς_0. Thus, we express the five-year abridged rates for the pooled model as (12).

$$
\begin{aligned}
{}_5q_{x,y,z,s} = 1 - \exp\left(-5\left(\alpha_0 e^{\alpha_1 y+\alpha_2 y^2+\alpha_3 z+\alpha_4 z^2+\alpha_5 s}\right)\right) \times \\
\exp\left(-5\left(-\beta_0 e^{\beta_1 y+\beta_2 y^2+\beta_3 s}\right)\right) \times \\
\left(\left(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2+\varsigma_3 s}+1\right)^5 - 1\right) \times \\
\left(exp\left(-\frac{\beta_0 e^{\beta_1 y+\beta_2 y^2+\beta_3 s}}{log\left(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2+\varsigma_3 s}+1\right)}\right)\right)^{\left(\varsigma_0 e^{\varsigma_1 y+\varsigma_2 y^2+\varsigma_3 s}+1\right)^x}
\end{aligned} \tag{12}
$$

The sampling model and joint posterior density are similar to the case of the independent model for each sex with a modification regarding the sex indicator and ratio parameters. The sampling model is given by

$$
({}_5Q_{x,y,z,s}|\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*, \boldsymbol{\varsigma}^*) \sim \text{beta}\left(1, \frac{1-{}_5q_{x,y,z,s}}{{}_5q_{x,y,z,s}}\right) \tag{13}
$$

where $\boldsymbol{\alpha}^* = \{\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}, \boldsymbol{\beta}^* = \{\beta_0, \beta_1, \beta_2, \beta_3\}$, and $\boldsymbol{\varsigma}^* = \{\varsigma_0, \varsigma_1, \varsigma_2, \varsigma_3\}$.

Furthermore, the posterior joint density is

$$
\begin{aligned}
\text{Posterior} \propto \alpha_0^{-0.675124} \beta_0^{-1} \varsigma_0^{-1} \times \\
e^{-\frac{1}{2}\left\{\sum_{i=1}^4 \left(\frac{\alpha_i}{5.45\times10^{-3}}\right)^2 + \sum_{i=1}^2 \left(\frac{\beta_i}{3.34\times10^{-3}}\right)^2 + \sum_{i=1}^4 \left(\frac{\varsigma_i}{1.96\times10^{-4}}\right)^2\right\}} \times \\
e^{-\frac{1}{2}\left\{\left(\frac{log\,\beta_0+13.997}{7.271}\right)^2 + \left(\frac{log\,\varsigma_0+2.536}{0.427}\right)^2\right\}} \times \\
e^{-\frac{1}{2}\left\{\left(\frac{\alpha_5}{7.824}\right) + \left(\frac{\beta_3}{3.524}\right)^2 + \left(\frac{\varsigma_3}{1.717\times10^{-1}}\right)^2\right\}} \times e^{-22.178\alpha_0^{0.325}} \times \\
\prod_{i=1}^{118}\left(\frac{\Gamma\left(\frac{1}{q_i}\right)}{\Gamma\left(\frac{1-q_i}{q_i}\right)}(q_i)^{\frac{1}{q_i}-1}(1-q_i)^{\frac{1}{q_i}-2}\right)
\end{aligned} \tag{14}
$$

We have 236 pieces of data in total to fit into this model, which consists of equal size of 118 data for males and females. The sample size ratio to the number of parameters is 16.9: 1. Since the ratio is greater than that of the independent sex model, this pooled model has more degrees of freedom so that we have more power to reject a false null hypothesis and find a significant result.

III. Results and Discussion

We obtained the parameter estimates in their posterior distributions by implementing our modeling approach. Their posterior means are further substituted into independent sex models in Equation 6 and pooled sexes model in Equation 12 to form the fitted model. The accuracy of the model is tested on both training and testing data. Furthermore, we analyzed the mortality trend to forecast future mortality tables and life expectancy.

A. Parameter Estimates for Independent Sex Models

We estimated the parameters using the R2WinBUGS package [34] in R version 3.6.0 with WinBUGS14 [33]. We determined which trend parameters are significant by evaluating their estimated 95% credible interval. To ensure the convergence of our estimates, we run two different chains for each sex, with 10,000 iterations for burn-in and 100,000 iterations for sampling in one chain. This process ran for a total of around two hours and ten minutes (for males), and 57 minutes (for females) on a computer with Intel® Core™ i5-8250U processor.

After applying single-chain convergence diagnostics such as the Heidelberger-Welch test (with ε of 2) and the Geweke test (p-value, of 0.01), we decided to burn another 10,000 iterations (for males) and kept all the 100,000 iterations (for females) or to achieve stationarity. We obtained our multivariate potential scale reduction factor values 1.02 (for males) and 1.00 (for females) in Gelman-Rubin multi-chain convergence diagnostic by considering the two chains. In the end, we combined results from the two chains for each sex to base on our statistical inference about the posterior distribution of the parameters.

A parameter is considerably significant (at the significance level of 0.01) if the 0.5-percentile to 99.5-percentile range does not contain zero. Examining the summary statistics, no significant trend parameters were found. Therefore, other statistics are required to understand the mortality trend. For this purpose, we introduce the use of posterior probabilities of the parameters to be negative (or positive), as it implies the non-zero value of those parameters, indicating the existence of trends.

TABLE I
POSTERIOR PROBABILITIES OF NEGATIVE TREND PARAMETERS FOR INDEPENDENT SEX MODELS

Parameter	Prob (Parameter < 0 | Data)	
	Males	Females
α_1	0.497	0.479
α_2	0.817	0.807
α_3	0.518	0.503
α_4	0.585	0.591
β_1	0.282	0.262
β_2	0.679	0.500
ς_1	0.460	0.347
ς_2	0.808	0.869

Examining the total number of kept iterations (i.e., the sample size), Prob(Parameter < 0 | Data) is considered to give us an insignificant result if its value lies between 0.496 and 0.504 (for males) or between 0.497 and 0.503 (for females), as these values imply that around 50:50 chance of being negative or the complement. Therefore, by looking at the numbers in Table I, all trend parameters except α_1 (for males) or α_3 and β_2 (for females) are significant to understand the mortality trend. Moreover, since the probability of all other parameters, except β_1 and ς_1 (for both sexes), to be negative are greater than 0.504 (for males) and 0.503 (for females), it implies these parameters represented the tendency of the negative trend of factors.

B. Accuracy Testing for Independent Sex Models

We obtained the fitted model by substituting the posterior means of the parameters into Equation 6. To assess the reliability and robustness of the accuracy of the fitted model, we use the model to forecast both training and testing data. As our accuracy metrics, we implemented mean absolute percentage error (MAPE) and coefficient of determination (R2). According to Gatabazi and Pindza [35], the MAPE criteria for accuracy are highly accurate, good, reasonable, and inaccurate for the range of under 10%, between 10% to 20%, between 20% to 50%, and >50%, respectively. In the whole training data, our model yields MAPE of 36.89% (for males) or 33.10% (for females) and R^2 of 98.57% (for males)

or 98.34% (for females), implying that the estimates are reasonably accurate.

The model was fit into data with $x \in [30, 80]$. The model's ability to extrapolate was assessed on the calculation of five-years abridged mortality rates for $x \in \{85, 90, 95\}$. We fitted our model for the population who are expected to be studied in the 2000-2005 period or before. For this criterion, the model yields MAPE of 2.65% (for males) and 5.16% (for females) and R^2 of 96.56% (for males) and 88.82% (for females), which imply that we could use the model for extrapolation purposes in term of age.

The model was then used to project the mortality rates across cohorts and periods. We have only fifteen data from five cohorts to assess forecasting accuracy by cohorts, so accuracy testing is done for all available data. We obtained MAPE of 45.93% (for males), and 6.10% (for females), and the resulting R^2 is 97.04% (for males) and 98.80% (for females). Moreover, we have three abridged life tables (each in 2005-2010, 2010-2015, and 2015-2020 study period) to assess forecasting accuracy by study periods, and the assessment is going to be done for each period. MAPE is in the range of [18.92%, 22.15%] for males and [8.20%, 8.54%] for females, when R^2 exceeds 99% for both males and females in all three study periods. Therefore, those results showed that our model could produce considerably high accuracy forecasts.

Moreover, we tried to determine our best estimate and worst estimate of mortality rates. By best estimates, we mean that it is the lowest logical probability of death, and the worst estimate means that it is the highest logical probability of death. For example, whenever our study resulted that the probability of a male aged 70 in 2030-2035 to die within the age of [70, 75) is between 2.549% and 48.33%, it is still possible that actual death is only 0.5%. We adjust the values based on historical data to suit the Indonesian males' society concerning their health background and other related risk factors. By substituting the i^{th}-percentile from posterior distributions of the parameters into the model as the best estimate and $(100\text{-}i)^{th}$-percentile as the worst estimate, with $i \in \{0, 1, 2, \ldots, 49\}$, we found that all values in training data fit for $i \leq 41$ (for both sexes) and all values in testing data for $i \leq 40$ (for males) or 39 (for females).

C. Mortality Trend Analysis by Using Independent Sex Models

The study period is represented by z variable and only affects the value of α through parameters α_3 and α_4 in our model. Our fitted model has negative values for both parameters, so it is expected that generally, mortality rates always decline concerning time. However, we also need to consider the values of Prob ($\alpha_3 < 0$ | Data) and Prob ($\alpha_4 < 0$ | data) as they are not far over 0.5 for both sexes. Further calculation shows that Prob ($\alpha_1 < 0$ and $\alpha_2 > 0$ | Data) = 0.210 (for males) or 0.209 (for females), implying that the possibility of having a turning point is statistically significant. It is also possible that actually mortality rates always increase with respect to time since the value of Prob ($\alpha_1 > 0$ and $\alpha_2 > 0$ | Data) = 0.205 (for males) and 0.200 (for females).

We used the birth year as cohort information, and the y variable represents it. Our fitted model has negative values of α_2, β_2, and ς_2, so it is expected that in the long term, the mortality rates decline with respect to a birth year for both sexes. By looking at positive-valued β_1 and ς_1, the values of α, B, and ς just start to go down together for those who were born in 1980 afterward (for males), or 1970 afterward (for females). However, we also need to prepare for another scenario that the mortality trend is going to have a turning point and will increase in the long term. This importance is suggested by the fact that the values of Prob ($\alpha_2 < 0$ | Data), Prob($\alpha_4 < 0$ | Data), Prob($\beta_2 < 0$ | Data), and Prob($\varsigma_2 < 0$ | Data) are all less than 0.99 for both sexes.

D. Comparing to The Results of Pooled Model

For every parameter in the independent sex models, all the symmetric 95% credible intervals for males and females overlap. Posterior probabilities of the trend parameters in both models to be negative are also considered close up to the first decimal digit (except β_2 and ς_1). Therefore, it is considered safe and plausible to assume those trend parameters for both sexes are equal, so we could proceed to use the pooled model. With a similar procedure as in using independent sex models, we also run two different chains, each chain with 10,000 iterations for burn-in and 100,000 iterations for sampling. The running time took two hours and 32 minutes in total on the same computer.

TABLE II
POSTERIOR PROBABILITIES OF NEGATIVE TREND PARAMETERS FOR POOLED MODEL

Parameter	Prob(Parameter < 0 | Data)
α_1	0.530
α_2	0.769
α_3	0.504
α_4	0.587
α_5	0.423
β_1	0.273
β_2	0.790
β_3	0.222

We decided to burn 15,000 more iterations to achieve the same convergence criteria defined for independent sex models. The multivariate potential scale reduction factor grows to 1.19, but it still satisfies convergence as univariate point estimates are lower than their respective upper limit of the confidence interval. By looking at the symmetric 95% credible interval of the ratio parameters, the difference in mortality rates between males and females is insignificant. It also suggests that the trend parameters are not significantly different from zero, so once again, we need to calculate posterior probabilities of the trend and ratio parameters to be negative as written in Table II. Each parameter is considered significant if significant if its respective probability is less than 0.496 or more than 0.504.

Table II suggests that all trend parameters (expect α_3) and ratio parameters are significant. Negative values for α_2, α_4, β_2, and ς_2 implies the expectation that the value of α, B, and ς decrease over time. Prob ($\alpha_5 < 0$ | data) and Prob ($\beta_3 < 0$ | data) are less than a half, then we expect that the value of α and B for males are lower than females. However, Prob ($\varsigma_3 < 0$ | Data) is greater than 0.5, so the value of ς is higher for males than females, and we found no significant different mortality rates by sex.

Implementing similar procedures as defined for independent sex models, the accuracy of the pooled model is

presented in Table III. The value of α equals 43 (for training data of males), 42 (for testing data of males), 41 (for training data of females), and 40 (for testing data of females). The pooled model gives the males better accuracy, and the independent sex model is better for females.

TABLE III
OVERALL ACCURACY ASSESSMENT FOR THE POOLED MODEL

Parameter	MAPE		R^2	
	Males	Females	Males	Females
Training data	34.95%	39.05%	98.26%	98.15%
Extrapolation to $x \in$ [85, 95]	3.42%	2.33%	97.21%	95.18%
Forecasting accuracy by cohorts (overall)	9.90%	13.48%	97.32%	98.78%
Forecasting accuracy to 2005-2010	9.93%	18.09%	99.34%	99.68%
Forecasting accuracy to 2010-2015	11.75%	17.58%	99.25%	99.67%
Forecasting accuracy to 2015-2020	15.79%	15.54%	99.23%	99.66%
Forecasting accuracy by time periods (overall)	12.49%	17.07%	99.23%	99.60%

Our fitted pooled model also provides decreasing mortality rates over the study period and cohort information, as the expected value of α_2, α_4, β_2, and ς_2 are all negative with significant probabilities. The declining trend of mortality rates provided by the pooled model is stronger than that provided by the independent sex models, as the estimated values of α, B, and ς decrease together for those who were born in 1915 afterward. The possibility to have a turning point in the mortality trend is still significant as the values of Prob ($\alpha_2 < 0$ | Data), Prob($\alpha_4 < 0$ | Data), Prob($\beta_2 < 0$ | Data), and Prob($\varsigma_2 < 0$ | Data) are all less than 0.99.

E. Forecasting Mortality Table

This section serves our research objective to forecast the abridged mortality table for the 2020-2025 period until the 2045-2050 period in the form of interval estimates, both by independent sex models and pooled models. Since the values of α are around 40 for both sexes and both models, our best estimate substituted 40-percentile values from posterior distributions of the parameters into the model, while the worst estimate substituted 60-percentile values. This procedure considers the approximation of predictive mortality rates instead of the posterior one. Due to maintaining the length of the resulting manuscript, complete mortality tables are not printed here.

F. Forecasting Life Expectancy

We have not concluded which approach is the best fit among independent sex models and pooled sex models. Outside of its accuracy, we also consider that the parameter estimates must be logical because it is neither too pessimistic nor too optimistic. Therefore, we need to calculate another indicator, and it is done by forecasting life expectancy. Assuming that we could extrapolate our model to $x \geq 100$, and the model is directly expressed in the form of μ_x, we could numerically calculate the value of complete life expectancy at age 30, symbolized as $\overset{o}{e}_{30}$.

One important thing to consider is, our function of μ_x is discontinuous, as birth year and study period are aggregated in five years. We also limit values of y and z to be integer-valued. These conditions work well for the calculation of ${}_tp_x$ with $t \leq 5$, because the value of y and z are constants in those calculations. Furthermore, we need to consider across study periods if we try to calculate ${}_tp_x$ with $t > 5$. Therefore, we need to define how to calculate them. For someone who is born in year w and will be studied in age x, with $w \geq 1950$, y and z are calculated as (15) and (16).

$$y = \frac{w-1855}{5} \tag{15}$$

$$z = \frac{5\frac{w+x}{5}-1950}{5} = \frac{w+x}{5} - 390 \tag{16}$$

We have to express the function of $\mu_{x,w,s}$, the force of mortality at age x for a male (s = 1) or a female (s = 0) who is born in year w. Recall that in the independent sex models, we have a different estimate of parameters for males and females. We have to look carefully for the respective sex and compute the hazard rates with (17).

$$\mu^{\text{ISM}}_{x,w,s} = \alpha_{0,s} e^{\alpha_{1,s}\frac{w-1855}{5}+\alpha_{2,s}\frac{w-1855^2}{5}+\alpha_{3,s}\left(\frac{w+x}{5}-390\right)+\alpha_{4,s}\left(\frac{w+x}{5}-390\right)^2} - \beta_{0,s} e^{\beta_{1,s}\frac{w-1855}{5}+\beta_{2,s}\frac{w-1855^2}{5}} + \left(\beta_{0,s} e^{\beta_{1,s}\frac{w-1855}{5}+\beta_{2,s}\frac{w-1855^2}{5}}\right) \times \left(1 + \varsigma_{0,s}\, exp\left\{\varsigma_{1,s}\frac{w-1855}{5} + \varsigma_{2,s}\frac{w-1855^2}{5}\right\}\right)^x \tag{17}$$

On the other side, if we implement the pooled model, parameter estimates are equal for males and females, but we have to consider the value of α_5. Therefore, the function of $\mu_{x,w,s}$ for the pooled model is written in (18).

$$\mu^{\text{PM}}_{x,w,s} = \alpha_0 e^{\alpha_1\frac{w-1855}{5}+\alpha_2\frac{w-1855^2}{5}+\alpha_3\left(\frac{w+x}{5}-390\right)+\alpha_4\left(\frac{w+x}{5}-390\right)^2+\alpha_5 s} - \beta_0 e^{\beta_1\frac{w-1855}{5}+\beta_2\frac{w-1855^2}{5}} + \left(\beta_0 e^{\beta_1\frac{w-1855}{5}+\beta_2\frac{w-1855^2}{5}+\beta_3 s}\right) \times \left(1 + \varsigma_0\, exp\left\{\varsigma_1\frac{w-1855}{5} + \varsigma_2\frac{w-1855^2}{5} + \varsigma_3 s\right\}\right)^x \tag{18}$$

1) Forecasting Life Expectancy Based on Independent Sex Models: By substituting posterior means of the parameters into the independent sex models, we obtained our expected value of $\overset{o}{e}_{30}$ for several birth years. The numbers show that the expected life expectancy is increasing for the younger cohort and if this declining mortality trend continues, an individual who was born in 2005 afterward (for males) or 1980 afterward (for females) could be expected to become centenarian after surviving the age of 30. Therefore, insurance companies must consider longevity risk well, especially for whole-life insurance and pension funds providers.

Referring to our best estimate of forecasting models, we plot our best estimate of $\overset{o}{e}_{30}$ values in Fig. 2. At their best, individuals who were born in 1990 afterward (for males) or 1975 afterward (for females) will probably exceed age 150 if they survive until age 30. Some parties may consider these results a wild dream, but it is possible by looking at lifespan records. Our models suggest that the expected total lifespan record, which we can see today in Indonesia, is in the range

of [87, 90] years (for males) or [91, 95] years (for females). This result aligns with the raw data used to generate the mortality table by the Indonesian Life Insurance Association, that the observed age was up to 110.

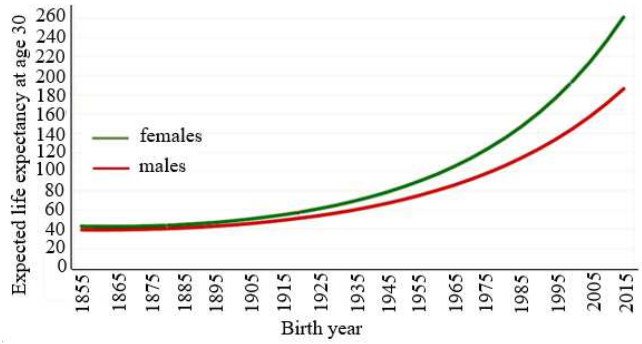

Fig. 2 Best estimate of life expectancy trend at age thirty, implied by the independent sex models.

Looking at another perspective, we have to give more attention to males' position as the breadwinner in their families. Especially when the family's income is only from them, males' roles in making a living for their families are very important, at least until their children complete formal education and no longer under their affordance. The mode of fathers' age when their children were born was 33, for the Jakarta population in 2015 [36]. Other data shows that most births in Jakarta occurred when the mothers' ages were in the range of [25, 29], as presented in Jakarta Open Data [37]. Assuming this condition also applies to other locations in Indonesia, we expect that most Indonesian children complete their 12-years compulsory education when their fathers and mothers are fifty and forty years old, respectively.

On the other side, the Indonesian government set the pension age at 57. Therefore, we need to calculate our worst estimate of probabilities that a 30 year old male passes away before his child completes compulsory education ($_{30}q_{20}$) and before he reaches pension age ($_{30}q_{27}$), respectively. Fig. 3 suggests that the probabilities are statistically significant at p-values of 0.01, 0.05, and 0.10 for all born in 2015 or before. Thus, it is suggested that having an insurance policy could be an advantage.

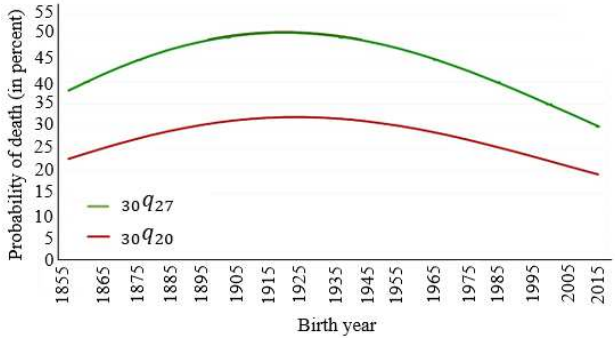

Fig. 3 Worst estimate of death probabilities trend for males, implied by the independent sex models.

Assuming that the father is dead when his child completes the compulsory education and his wife substitutes his role to be a breadwinner at least until the child obtains an undergraduate degree, we consider that it is important to calculate the worst estimates of $_{40}q_4$ and $_{40}q_{17}$ (i.e., the probability that the mother dies before her child obtains undergraduate degree, and before she reaches pension age, respectively). The estimated values of $_{40}q_{4\ range}$ between 20% to 35%; and between 30% to 50% for $_{40}q_{17}$. These probabilities are still considered high, as they are statistically significant (p-value = 0.01) for all females born in 2015 or before. Therefore, it is also suggested that married females have life insurance policies.

2) Forecasting Life Expectancy Based on Pooled Model: We also calculated all statistics that we have considered similarly for independent sex models. Our pooled model suggested that females have a longer lifespan than males, but males still live longer by looking at the best estimates. Based on these two scenarios, the pooled model gives a more optimistic estimate for the males and a more pessimistic estimate for the females than the independent sex models in the long term.

However, the pooled model gives a less optimistic estimate for the males and a more optimistic estimate for the females for the worst scenario. We chose not to provide detailed results here because this article's length is reasonable, but readers could look for them as separate supplementary material. Our intuitive explanation and conclusion for the results of this pooled model are similar to the independent sex models; we have to seriously consider both longevity risk and mortality risk.

IV. Conclusion

We found that mortality rates declined with time, with newer data suggesting slowing the decline. Our study proposed an eleven-parameters Makeham model considering both birth year and study period. We implemented the Bayesian method to forecast mortality rates of the Indonesian old-age population. We also calculated the probabilities that the decline would stop and turn into increasing mortality rates in the future. Our results expect that mortality rates are declining in the long term. Insurance companies need to consider both mortality risk and longevity risk to ensure their profitability and financial health in the future. The probability of a turning point is statistically significant, so it is recommended to be aware of the possibility of worse mortality rates in the future.

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References

[1] F. Sari, "Minat masyarakat bergeser, kontribusi premi produk asuransi jiwa tradisional turun," *Kontan.co.id*, Jul. 08, 2019. https://keuangan.kontan.co.id/news/minat-masyarakat-bergeser-kontribusi-premi-produk-asuransi-jiwa-tradisional-turun (accessed Jan. 21, 2021).

[2] D. Dickson, M. Hardy, M. Hardy, and H. Waters, *Actuarial Mathematics for Life Contingent Risks*, 2nd ed. New York: Cambridge University Press, 2018.

[3] M. H. Hsieh, C. J. Tsai, and J. L. Wang, "Mortality Risk Management Under the Factor Copula Framework—With Applications to Insurance

Policy Pools," *North Am. Actuar. J.*, vol. 25, no. S1, pp. S119–S131, 2021, doi: 10.1080/10920277.2019.1653201.
[4] R. MacMinn and A. Richter, "The choice of trigger in an insurance linked security: the mortality risk case," *Insur. Math. Econ.*, vol. 78, pp. 174–182, 2018.
[5] T. Andersen and M. Gestsson, "Annuitization and aggregate mortality risk," *J. Risk Insur.*, vol. 88, no. 1, pp. 79–99, 2021.
[6] A. Wibisana and M. Novita, "Simulation of mortality immunization for life insurance companies in Indonesia using duration and convexity approach," *J. Phys. Conf. Ser.*, vol. 1725, p. 12082, 2021, doi: 10.1088/1742-6596/1725/1/012082.
[7] M. D. Schmeck and H. Schmidli, "Mortality options: the point of view of an insurer," *Insur. Math. Econ.*, vol. 96, pp. 98–115, 2021.
[8] S. Both, V. Horneff, B. Kaschützke, and R. Maurer, "Surplus participation schemes for life annuities under Solvency II," *Eur. Actuar. J.*, vol. 9, no. 2, pp. 391–421, Dec. 2019, doi: 10.1007/s13385-019-00203-3.
[9] S. H. Cox, Y. Lin, and S. Liu, "Optimal longevity risk transfer and investment strategies," *North Am. Actuar. J.*, vol. 25, no. S1, pp. S40–S65, 2021, doi: 10.1080/10920277.2019.1692617.
[10] T. Moenig, "Variable annuities: Market incompleteness and policyholder behavior," *Insur. Math. Econ.*, vol. 99, pp. 63–78, 2021.
[11] A. R. Bacinello, P. Millossovich, and A. Chen, "The Impact of longevity and investment risk on a portfolio of life insurance liabilities," *Eur. Actuar. J.*, vol. 8, no. 2, pp. 257–290, 2018.
[12] D. Blake, A. J. G. Cairns, K. Dowd, and A. R. Kessler, "Still living with mortality: the longevity risk transfer market after one decade," *Br. Actuar. J.*, vol. 24, pp. 1–80, 2019, doi: 10.1017/S1357321718000314.
[13] M. A. Mlevsky, "Calibrating Gompertz in reverse: What is your longevity-risk-adjusted global age?," *Insur. Math. Econ.*, vol. 92, pp. 147–161, 2020.
[14] H.-C. Wang, C. S. J. Yue, and C. T. Chong, "Mortality models and longevity risk for small populations," *Insur. Math. Econ.*, vol. 78, pp. 351–359, 2018.
[15] A. Kessler, "New Solutions to an Age-Old Problem: Innovative Strategies for Managing Pension and Longevity Risk," *Taylor Fr.*, vol. 25, no. S1, pp. S7–S24, 2019, doi: 10.1080/10920277.2019.1672566.
[16] N. Bugler, K. Maclean, V. Nicenko, and P. Tedesco, "Reinsurance Sidecars: The Next Stage in the Development of the Longevity Risk Transfer Market," *North Am. Actuar. J.*, vol. 25, no. S1, pp. S25–S39, 2021, doi: 10.1080/10920277.2019.1673183.
[17] M. Roser, E. Ortiz-Ospina, and H. Ritchie, "Life expectancy," *Our World in Data*, 2019. https://ourworldindata.org/life-expectancy (accessed Dec. 21, 2020).
[18] A. Case and A. Deaton, "Mortality and morbidity in the 21st century," *Brookings Pap. Econ. Act.*, p. 397, 2017.
[19] C. E. Chandra and S. Abdullah, "Estimating Indonesian complete life table and fair annual pure premium range from abridged life table with Bayesian method and bootstrapping." Under review in a peer-reviewed journal, 2021.
[20] United Nations, "World population prospects - population division - United Nations," *Department of Economic and Social Affairs Population Dynamics*, 2020. https://population.un.org/wpp/default.aspx?aspxerrorpath=/wpp/Download/Standard/Mortality/WPP2019_MORT_F17_3_ABRIDGED_LIFE_TABLE_FEMALE.xlsx. (accessed Oct. 02, 2020).
[21] United Nations, "World population prospects - population division - United Nations," *Department of Economic and Social Affairs Population Dynamics*, 2020. https://population.un.org/wpp/default.aspx?aspxerrorpath=/wpp/Download/Standard/Mortality/WPP2019_MORT_F17_2_ABRIDGED_LIFE_TABLE_MALE.xlsx. (accessed Oct. 01, 2020).
[22] F. Janssen, "Advances in mortality forecasting: introduction," *Genus*, vol. 74, no. 1. Springer International Publishing, pp. 1–12, Dec. 01, 2018, doi: 10.1186/s41118-018-0045-7.
[23] A. Hunt and D. Blake, "On the structure and classification of mortality models," *North Am. Actuar. J.*, vol. 25, no. S1, pp. S215–S234, 2020, doi: 10.1080/10920277.2019.1649156.
[24] L. Safitri, S. Mardiyati, and H. Rahim, "Forecasting the mortality rates of Indonesian population by using neural network," *J. Phys.*, vol. 974, no. 1, Mar. 2018, doi: 10.1088/1742-6596/974/1/012030.
[25] Y. Dong, F. Huang, H. Yu, and S. Haberman, "Multi-population mortality forecasting using tensor decomposition," *Scand. Actuar. J.*, vol. 2020, no. 8, pp. 754–775, Sep. 2020, doi: 10.1080/03461238.2020.1740314.
[26] C. Qiao and M. Sherris, "Managing systematic mortality risk with group self-pooling and annuitization schemes," *J. Risk Insur.*, vol. 80, no. 4, pp. 949–974, Dec. 2013, doi: 10.1111/j.1539-6975.2012.01483.x.
[27] J. Hilton, E. Dodd, J. J. Forster, and P. W. F. Smith, "Projecting UK mortality by using Bayesian generalized additive models," *J. R. Stat. Soc. Ser. C Appl. Stat.*, vol. 68, no. 1, pp. 29–49, Jan. 2019, doi: 10.1111/rssc.12299.
[28] H. K. Kindsvater, N. K. Dulvy, C. Horswill, M.-H. Juan-Jorda, M. Mangel, and J. Matthiopoulos, "Overcoming the data crisis in biodiversity conservation," *Trends Ecol. Evol.*, vol. 33, no. 9, pp. 676–688, 2018.
[29] M. Miočević, D. P. Mackinnon, and R. Levy, "Power in Bayesian Mediation Analysis for Small Sample Research," *Struct. Equ. Model. A Multidiscip. J.*, vol. 24, no. 5, pp. 666–683, Sep. 2017, doi: 10.1080/10705511.2017.1312407.
[30] A. Gelman, D. Simpson, and M. Betancourt, "The Prior Can Often Only Be Understood in the Context of the Likelihood," *Entropy*, vol. 19, p. 555, 2017, doi: 10.3390/e19100555.
[31] S. C. Smid, D. McNeish, M. Miocevic, and R. van de Schoot, "Bayesian versus frequentist estimation for structural equation models in small sample contexts: a systematic review," *Struct. Equ. Model. A Multidiscip. J.*, vol. 27, no. 1, pp. 131–169, 2019.
[32] C. W. Jordan, *Society of Actuaries' Textbook on Life Contingencies*. Chicago: The Society of Actuaries, 1991.
[33] D. J. Lunn, A. Thomas, N. Best, and D. Spiegelhalter, "WinBUGS - A Bayesian modelling framework: Concepts, structure, and extensibility," *Stat. Comput.*, vol. 10, no. 4, pp. 325–337, 2000, doi: 10.1023/A:1008929526011.
[34] S. Sturtz, U. Ligges, and A. Gelman, "R2WinBUGS: A Package for Running WinBUGS from R," 2005. Accessed: Jun. 21, 2020. [Online]. Available: http://www.jstatsoft.org/.
[35] P. J. C. M. Gatabazi and E. Pindza, "Modeling cryptocurrencies transaction counts using variable-order Fractional Grey Lotka-Volterra dynamical system," *Solitons & Fractals*, vol. 127, pp. 283–290, 2019.
[36] Jakarta Open Data, "Data jumlah kelahiran berdasarkan usia ayah," *Jakarta Open Data*, Jul. 18, 2018. https://data.jakarta.go.id/dataset/jumlah-kelahiran-berdasarkan-usia-ayah-tahun-2015 (accessed Jun. 21, 2020).
[37] Jakarta Open Data, "Data kelahiran bayi berdasarkan usia ibu," *Jakarta Open Data*, Jun. 22, 2018. https://data.jakarta.go.id/dataset/data-kelahiran-bayi-berdasarkan-usia-ibu-melahirkan-tahun-2017/resource/58c1f680-bae3-4a61-ab9d-88ccdbb22a99. (accessed Jun. 21, 2020).