Abstract—High-strength concrete is one of the various special concretes that have become increasingly popular in recent decades. High-strength concrete offers a higher strength-to-volume ratio than normal-strength concrete. However, the design provision is not explicitly served in most building codes. This study focuses on the shear strength of high-strength concrete and one of many factors that influence the shear strength, i.e., the longitudinal reinforcement ratio. The influence of the longitudinal reinforcement ratio was analyzed and compared with twelve high-strength reinforced concrete beams without coarse aggregate. Concretes with cylinder compressive strengths ranging from 58 to 110 MPa were used. The concrete mixes were made without coarse aggregate, with the maximum aggregate size of #30 sieve. The beam specimens were reinforced with various longitudinal reinforcement ratios and were tested until failure using a four-point bending test setup. The tests showed that the degree of influence of longitudinal reinforcement was in agreement with the Eurocode 2 (EC2) formula, but the formula overestimated the concrete's shear strength. Based on the results, a modification was then proposed to the existing formula to improve its accuracy for high-strength concrete. The modified formula significantly improves shear strength prediction accuracy compared to the existing EC2-2004 and the formulas by other researchers for specimens used in this research. Due to the limited number of specimens used in this research, future research could be done to verify the resulting modified equation and generalize it for a wider range of concrete strength and section shape.

Keywords—Coarse aggregate; Eurocode 2; high-strength concrete; longitudinal reinforcement; shear strength.

I. INTRODUCTION

Concrete is one of the most used materials in construction, with its usage dating back to roughly 2000 BC. In recent years, there have been numerous types of research to improve conventional concrete, either its mechanical properties, economy, or other properties that are desirable for certain conditions [1]–[5]. One of them is high-strength concrete. Generally, concrete with compressive strength above 55 MPa is considered high-strength concrete. However, high-strength concrete is not explicitly defined in Eurocode 2, but concrete with a nominal cylinder strength of 50 MPa to 70 MPa can be classified under the high strength class [6].

High-strength concrete is commonly used for the construction of high-rise buildings. Columns made from high-strength concrete can support higher loads and are smaller than their normal strength counterparts [7],[8]. Properties of high-strength concrete commonly researched in present days include its behavior when subjected to shear, axial strength, water absorption, etc. [9]–[16] and also its mixtures [17]–[19]. High-strength concrete mix is usually made with low w/c, in the range of 0.40 or lower. In order to make concrete with low w/c and acceptable workability, supplementary materials such as superplasticizer and pozzolan are required in the mix [20]. As with conventional, normal-strength concrete, beams of high-strength concrete are still subjected to bending moment and shear [21]–[24]. Shear in reinforced concrete beams without stirrup triggers cracks on inclined planes, especially near the supports, as presented in Fig. 1.

The diagonal tensile stress gives rise to inclined cracks. This shear is resisted by the beam and arch mechanisms and will eventually fail when those mechanisms are no longer capable of transferring the shear forces [26]. The shear failure mechanism is complicated and depends on the ratio of shear span \( a_v \) to effective section depth \( d \). Shear span \( a_v \) is determined as the space between the support and point load working on the span.
Previous researches suggest that shear strength and transfer of reinforced concrete beams without transverse reinforcement is influenced by several variables and mechanisms, such as shear interface type and area, aggregate size (aggregate interlock), and reinforcement ratio and strength (dowel action) [27]. Longitudinal reinforcement contributes to shear strength in various ways, but it mainly restrains the cracks' width and transfers shear force through dowel action. Increasing the longitudinal reinforcement ratio \( \rho \) can restrain the crack width. Inhibiting the propagation of flexural cracks and further increasing shear capacity carried through dowel action [19].

The bond between concrete and longitudinal reinforcement could also affect the shear strength. When \( w/c \) ratio is around 0.4, the strength of the bond between mortar is equivalent to that of the coarse aggregate [28]. Hence high-strength concrete is usually not coarse concrete because the strength of the coarse aggregate is sometimes lower than the cementitious matrix.

The exact evaluation of the contribution of each mechanism is difficult. Thus, these actions are generally lumped together as concrete shear strength. The contribution of the longitudinal reinforcement lies in many factors, such as the diameter and distribution of the steel bars [6]. Thus, concrete building codes, such as ACI 318 and Eurocode 2, based the contribution of longitudinal reinforcement solely on the longitudinal reinforcement ratio \( \rho \), without considering its configuration or size.

The longitudinal reinforcement ratio is generally given with an exponent, which is determined from the experiment. These exponents thus determined said formulations' assumption of longitudinal reinforcement's contribution to shear strength. Various building codes and researches have different values for these exponents. For example, Eurocode 2 2004 assumes shear strength is proportional to \((100\rho)^{0.5} \) [29], ACI 318M-19 assumes \( \rho^{0.9} \) [25], and other studies, such as by Kim and Park [30], assume \( \rho^{3.8} \).

As of Eurocode 2 2004, high-strength concrete has no special provisions. The nominal cylinder compressive strength is also limited to 90 MPa for design purposes, and high-strength concrete can easily exceed this limit. This research aims to analyze and compare the degree of contribution of longitudinal reinforcement to concrete shear strength on high-strength concrete using the existing Eurocode 2 formula. Based on the results, this research also proposes a modification to existing Eurocode 2 to estimate shear the strength of high-strength concrete used in this research with better accuracy.

### A. Shear Strength Formula for High-strength concrete

1) Eurocode 2 Shear Strength Formula: As of Eurocode 2 2004, there has not been a specific article or formula for predicting the shear strength of high-strength concrete. Eurocode 2 uses empirical equations that are believed to have considered the main factors that influence shear strength. By substituting the partial safety factor for the concrete, \( \gamma_c = 1.0 \), the concrete shear strength \( V_{Rd,c} \) can be expressed by the equation:

\[
VRd,c = 0.18k(100\rho f_{ck})^{1/3}bw_d \tag{1}
\]

Where:
- \( k \): size effect modification factor
- \( \rho \): longitudinal reinforcement ratio
- \( f_{ck} \): characteristic cylinder compressive strength
- \( k \): size effect modification factor = 1 + \((200/d)^{1/2} \leq 2\)
- \( d \): depth of the section

2) Formula from Various Researchers: The formula used for comparison is taken from various sources and listed in TABLE I. Some formulas are converted from ultimate shear stress to ultimate shear strength by multiplying them with cross-sections.

### TABLE I

<table>
<thead>
<tr>
<th>Source</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zsutty [31]</td>
<td>( V_c = 2.2(\rho f_{ck}d/a)^{1/3} b_{cd}, a/d \geq 2.5 ) \tag{2}</td>
</tr>
<tr>
<td>Bazant and Sun [32]</td>
<td>( V_c = 0.54\sqrt{\rho} \left( f_{ck} + 249 \rho/(b_d)^{5/2} \right) \left( 1 + 5.08 \rho^{1/3}/d \right) \left( 1 + \frac{d}{25d_c} \right) b_{cd} ) \tag{3}</td>
</tr>
<tr>
<td>Kim and Park [30]</td>
<td>( V_c = 3.5f_{ck} \rho^{0.75}(0.4 + d/a)\left(1/\sqrt{1+0.008d}+0.18\right)b_{cd} ) \tag{4}</td>
</tr>
<tr>
<td>Cavagnis [33]</td>
<td>( V_c = \kappa \left( 100\rho f_{ck} d_c \right)^{1/3} b_{cd} ) \tag{5}</td>
</tr>
</tbody>
</table>

### II. MATERIALS AND METHODS

In this research, the specimen data were taken from Christianto et al. [34]–[36]. The experimental outline is briefly discussed in the following sections. The specimens consist of twelve \( \phi 10 \text{ cm} \times 20 \text{ cm} \) cylinders to measure the compressive strength and twelve \( 7 \times 12.5 \times 110 \text{ cm} \) beams for the four-point bending test.
A. Materials and Mix Design

The materials used and their proportion for the concrete mix are summarized in TABLE II. Due to its fine particle size, silica fume is used as pozzolan material and fills the gap between mortar and aggregate [37].

<table>
<thead>
<tr>
<th>Materials</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary Portland Cement (OPC)</td>
<td>-</td>
</tr>
<tr>
<td>Silica sand (sieve no. 30 and no. 50)</td>
<td>110% of cement mass</td>
</tr>
<tr>
<td>Silica fume</td>
<td>20% of cement mass</td>
</tr>
<tr>
<td>Marble powder (sieve no. 200)</td>
<td>10% of cement mass</td>
</tr>
<tr>
<td>Superplasticizer</td>
<td>2.5% of cement mass</td>
</tr>
<tr>
<td>Accelerator</td>
<td>5 litres/m³ of concrete mix</td>
</tr>
</tbody>
</table>

B. Preparations of Specimens

Twelve specimens of $7 \times 12.5 \times 110$ cm$^3$ concrete beams were cast for testing. Each beam was provided with two rebar bars of various sizes, which are $\phi 6$, $\phi 8$, $\phi 10$, $\phi 12$, and $\phi 19$. Twelve $\phi 10$ cm $\times$ 20 cm cylinders were also cast to determine the cylinder strength. The concrete mix was mixed in a concrete mixer and then cast into the rectangular beam molds and cylinder molds. The molds were disassembled 48 hours after casting. Then the specimens were cured in a water bath for 58 days. After 58 days, the specimens were removed from the water bath and steam cured for 8 hours.

After the specimens were cured, the cylinder specimens were used for the compressive test, and the beam specimens were used for the four-point bending test. The model of the beam specimen can be seen in Fig. 2, and the photograph of both flexural and shear failure can be seen in Fig. 3. The details of each beam specimen are provided in TABLE III.

![Fig. 2 Model of the beam specimen [34]](image)

![Fig. 3 (a) Flexural failure, (b) Shear failure [34]](image)

III. RESULTS AND DISCUSSION

To observe the contribution of the longitudinal reinforcement ratio in Eurocode 2-2004 formula, the non-dimensional ratio, $\frac{V_{d,c}}{b_d d f_{ck}}^{1/3}$, is plotted against the cube root of longitudinal reinforcement ratio, $\rho^{1/3}$, to accommodate its contribution. $V_{d,c}$ is the concrete's shear strength. Non-dimensional ratio $\frac{V_{d,c}}{b_d d f_{ck}}^{1/3}$ is used to normalize the shear strength due to the variability of concrete compressive strength. Because Eurocode 2 2004 has a limitation on $\rho$ where $\rho \leq 2\%$, $V_{d,c}$ would be calculated both with and without said limitation. The concrete compressive strength maximum limit of 90 MPa is still used because the compressive strength is not interesting in this research. The plot between the non-dimensional ratio against $\rho^{1/3}$ is shown in Fig. 4. Trendlines are added to each plot to improve the legibility of the graph.

![Fig. 4](image)

Fig. 4 shows three plots of the cube root of longitudinal reinforcement ratio, $\rho^{1/3}$, versus non-dimensional ratio $\frac{V_{d,c}}{b_d d f_{ck}}^{1/3}$: actual shear strength from test results, shear strength prediction using Eurocode 2 2004 formula with the longitudinal reinforcement ratio limit, and shear strength prediction using Eurocode 2 2004 formula without the longitudinal reinforcement ratio limit. The cube root of the longitudinal reinforcement ratio is intended to make it easier to identify the fitness of the formula prediction against the test results. A linear plot with a similar gradient to actual shear strength means more fitness against the test results. The coefficient of determination ($R^2$) is also computed for the actual shear strength plot to determine the proportionality between $\rho^{1/3}$ and shear strength.

Based on Fig. 4, it can also be seen that there is an increase in the concrete shear strength as the longitudinal reinforcement ratio increases, but to different degrees. The gradient of the "test" slope is more similar to the Eurocode 2 2004 without the $\rho$ limitation rather than the original Eurocode 2 2004 formula with the limitation. The longitudinal reinforcement still has a considerable influence on shear strength even when it is greater than 0.02. Although Fig. 4 suggests that concrete shear strength could be increased by adding more longitudinal reinforcement, it is not practical and not economical.

![TABLE III SPECIMEN DATA [34]](image)
For checking the proportionality, the coefficient of determination ($R^2$) for the "test" plot is computed and has a value of 0.8354, which indicates fairly good linearity between $\rho^{1/3}$ and concrete shear strength. The actual exponent value may differ, but it can be deduced that the actual value is not differed by much to $1/3$.

The accuracy of shear strength prediction can also be seen in Fig. 4. The Eurocode 2 2004 formula, with and without the reinforcement ratio limit, overestimates the shear strength considerably. The formula with the reinforcement ratio limit only has an accurate prediction if the reinforcement ratio is around or greater than 0.25, and this condition is seldom found in practical cases. Meanwhile, the formula without the reinforcement limit overestimates the shear strength for all reinforcement ratio values. These overestimations may be caused by the absence of coarse aggregate, which may reduce the contribution of aggregate interlock, which resists slippage between concrete sections near the cracks.

With the shear strength known to be proportionally close to $\rho^{1/3}$, calculations are made to obtain the actual value of the exponent. The exponent value of 1/3 in the Eurocode 2 2004 formula is determined by parameter $r$. The value of $r$ is determined by calculation. Because shear strength is usually assumed to be proportional to $(100\rho)^{1/3}$ (such as in Eurocode 2 2004 formula) or $\rho^{1/3}$ (such as in ACI 318M-19 formula), the value of the exponential value $r$ can be calculated for both cases.

By replacing the exponent value of 1/3 with parameter $r$, the Eurocode 2 2004 formula for concrete shear strength can be written as Eq. (6) and (7).

$$V_{rd,c} = 0.18k(100\rho)^{1/3}b_wd$$  \hspace{1cm} (6)  

$$V_{rd,c} = 0.18k(100\rho r)^{1/3}b_wd$$  \hspace{1cm} (7)

By using the logarithm, Eq. (6) and Eq. (7) can be rearranged to isolate the variable $r$.

$$r = \log_{100\rho} \frac{V_{rd,c}}{0.18k^{1/3}b_wd}$$  \hspace{1cm} (8)  

$$r = \log_{0.18k^{1/3}b_wd} \frac{V_{rd,c}}{V_{rd,c}}$$  \hspace{1cm} (9)

Eq. (8) and Eq. (9) is the rearranged form of Eq. (6) and Eq. (7), respectively. Based on the previous discussion, the limitation on the reinforcement ratio is omitted in the calculation of the $r$-value. The calculated value of $r$, along with its average and coefficient of variation (CoV), are presented in Table IV. CoV is included to measure the scatter of the calculated value, where lower numbers indicate a more narrowly scattered, more consistent value.

### Table IV

**Exponential Value $r$**

<table>
<thead>
<tr>
<th>Beam</th>
<th>$\rho$</th>
<th>Shear strength (kN)</th>
<th>$r$</th>
<th>$r$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A21</td>
<td>0.77%</td>
<td>5.1655</td>
<td>2.4298</td>
<td>0.4463</td>
<td></td>
</tr>
<tr>
<td>A22</td>
<td>0.77%</td>
<td>5.0755</td>
<td>2.8381</td>
<td>0.4683</td>
<td></td>
</tr>
<tr>
<td>A11</td>
<td>1.37%</td>
<td>7.1205</td>
<td>-1.4701</td>
<td>0.4649</td>
<td></td>
</tr>
<tr>
<td>A12</td>
<td>1.37%</td>
<td>9.2855</td>
<td>-0.5805</td>
<td>0.4000</td>
<td></td>
</tr>
</tbody>
</table>

Based on values presented in Table IV, the average value of $r$ from Eq. (9), which assumes shear strength is proportional to $(100\rho)^r$, is close to the original value (0.3265 compared to 1/3 = 0.3333 on the original formula). While the average value is close to the original value, the CoV suggests otherwise. The CoV for values of $r$ calculated from Eq. (9) is 3.4570, or over 300%, which suggests the values are widely and wildly scattered. This scatters confirmed by looking into the individual value of $r$, which varies widely from -1.4701 to 2.4298. Due to the large scatter, Eq. (7), the original form of Eq. (9), is not suitable for predicting concrete shear strength.

On the other hand, values of $r$, which were calculated from Eq. (8) are more consistent and narrowly scattered as evidenced by a relatively far lower CoV value of 0.0969 or just under 10%. Equation (8) suggests that the exponential value is more suitable to be increased from 1/3 = 0.3333 to 0.4422 to predict shear strength for high-strength concrete more accurately. For simplicity, the average value of 0.4422 is approximated to be a fraction 4/9. Thus, the modified Eurocode 2 2004 formula to predict shear strength for high-strength concrete is:

$$V_{rd,c} = 0.18k(100\rho r)^{4/9}b_wd$$  \hspace{1cm} (10)

However, this result should not be generalized due to this research’s limited number of specimens. The value of $r$ can also be used to deduce the relative influence of longitudinal reinforcement on shear strength in high-strength concrete compared to normal-strength concrete. Since the value of the reinforcement ratio is always smaller than unity, a larger exponential value will result in a smaller value. The proposed exponential value of 4/9 is larger than the original value of 1/3. This indicates that the longitudinal reinforcement's influence is smaller in high-strength concrete than in normal-strength concrete. Particularly for no-coarse aggregate concrete.
The shear strength of each beam specimen is calculated using the proposed formula given by Eq. (10) and then compared to the results obtained using various formulas proposed by other researchers. The comparisons are listed in TABLE V. The comparisons are made using the ratio between actual shear strength from the test and predicted shear strength from the formula $V_{\text{test}}/V_{\text{form}}$. The average of the $V_{\text{test}}/V_{\text{form}}$ ratio and coefficient of variation (CoV) for each formula are tabulated in TABLE V.

Based on TABLE V, the modified Eurocode 2 formula gives the most accurate prediction of shear strength for high-strength concrete with an error of an average of just 3%. However, formulas by Zsutty [31] and Bazant and Sun [32] also have reasonable accuracy but are unconservative. On the other hand, the original Eurocode 2 formula [29] and formulas by Kim and Park [30] overestimate the shear strength, with the predicted strength almost double the actual strength. Conversely, the formula by Cavagnis [33] heavily underestimates the shear strength. This might be due to the formula that considers the effect of aggregate size, which is only fine-grain in the research. Most formulas give similar scatter with the CoV varying from 0.12 to 0.15.

### TABLE V

<table>
<thead>
<tr>
<th>Beam</th>
<th>$\rho$</th>
<th>Eq. (10)</th>
<th>Eq. (1) [29]</th>
<th>Eq. (2) [31]</th>
<th>Eq. (3) [32]</th>
<th>Eq. (4) [38]</th>
<th>Eq. (5) [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A21</td>
<td>0.77%</td>
<td>0.9542</td>
<td>0.5771</td>
<td>0.7120</td>
<td>0.8371</td>
<td>0.5810</td>
<td>2.1698</td>
</tr>
<tr>
<td>A22</td>
<td>0.77%</td>
<td>0.8906</td>
<td>0.5186</td>
<td>0.6398</td>
<td>0.7283</td>
<td>0.5220</td>
<td>1.9495</td>
</tr>
<tr>
<td>A11</td>
<td>1.37%</td>
<td>0.9158</td>
<td>0.5685</td>
<td>0.7013</td>
<td>0.7615</td>
<td>0.5587</td>
<td>2.1372</td>
</tr>
<tr>
<td>A12</td>
<td>1.37%</td>
<td>1.2101</td>
<td>0.7511</td>
<td>0.9267</td>
<td>1.0107</td>
<td>0.7382</td>
<td>2.8239</td>
</tr>
<tr>
<td>A31</td>
<td>2.14%</td>
<td>1.2211</td>
<td>0.7965</td>
<td>0.9259</td>
<td>0.9458</td>
<td>0.7240</td>
<td>2.8214</td>
</tr>
<tr>
<td>A32</td>
<td>2.14%</td>
<td>1.2818</td>
<td>0.8361</td>
<td>0.9636</td>
<td>0.9815</td>
<td>0.7535</td>
<td>2.9364</td>
</tr>
<tr>
<td>A41</td>
<td>3.08%</td>
<td>0.9524</td>
<td>0.6469</td>
<td>0.7981</td>
<td>0.8292</td>
<td>0.6146</td>
<td>2.4320</td>
</tr>
<tr>
<td>A42</td>
<td>3.08%</td>
<td>1.0857</td>
<td>0.7374</td>
<td>0.9098</td>
<td>0.9562</td>
<td>0.7007</td>
<td>2.7724</td>
</tr>
<tr>
<td>A51</td>
<td>5.47%</td>
<td>1.1727</td>
<td>0.8491</td>
<td>1.0476</td>
<td>1.0265</td>
<td>0.7877</td>
<td>3.1922</td>
</tr>
<tr>
<td>A52</td>
<td>5.47%</td>
<td>0.9052</td>
<td>0.6554</td>
<td>0.8086</td>
<td>0.8102</td>
<td>0.6080</td>
<td>2.4641</td>
</tr>
<tr>
<td>A61</td>
<td>7.72%</td>
<td>0.8859</td>
<td>0.6665</td>
<td>0.8222</td>
<td>0.7797</td>
<td>0.6094</td>
<td>2.5056</td>
</tr>
<tr>
<td>A62</td>
<td>7.72%</td>
<td>0.8331</td>
<td>0.6267</td>
<td>0.7732</td>
<td>0.7338</td>
<td>0.5731</td>
<td>2.3561</td>
</tr>
<tr>
<td>Average</td>
<td>1.0288</td>
<td>0.6858</td>
<td>0.8357</td>
<td>0.8667</td>
<td>0.6476</td>
<td>2.5467</td>
<td></td>
</tr>
<tr>
<td>CoV</td>
<td>0.1461</td>
<td>0.1508</td>
<td>0.1387</td>
<td>0.1222</td>
<td>0.1303</td>
<td>0.1387</td>
<td></td>
</tr>
</tbody>
</table>

### IV. CONCLUSION

Based on the results of this research for high-strength concrete without coarse aggregate with concrete cylinder compressive strength varies from 58 to 110 MPa, the authors conclude as follows: The Eurocode 2 2004 formula without limitations on $\rho$ gives a trendline with good agreement with the test results. The calculated values of $r$ vary from 0.3799 to 0.5157, with an average of 0.4422. All of the calculated values of $r$ are higher than the original formulas, which indicates the degree of contribution of longitudinal reinforcement is less significant than the Eurocode 2 2004 formula's assumption. The shear strength of high-strength concrete is proportional to $\rho^{0.9}$

### ACKNOWLEDGMENT

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