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Assessment of Damage in Prismatic Beams Using Modal Parameters

Pallab Das, Anjan Dutta, Sudip Talukdar

Civil Engineering Department, National Institute of Technology Silchar, Assam, 788010, India
E-mail: pallabdas9@gmail.com, adutta@iitg.ernet.in, staluk@iitg.ernet.in

Abstract—**In this study, the modal parameters of un-cracked and cracked plain cement concrete prismatic beams have been studied. A crack in a beam element introduces considerable local flexibility, which has been expressed by local flexibility matrix, the dimension of which depends upon the numbers of degree of freedom considered. An approach based on linear fracture mechanics theory has been used to find flexibility matrix for the cracked element. The FEM program has been developed for eigen-value problems to determine the modal parameters of the cracked beams. Numerical studies are performed by considering simply supported beam with single and multiple cracks at different locations with different crack depths. The changes in displacement mode shapes are not localized in the region of damage and hence, they do not give indication of the location of damage very precisely. With curvature mode shape, the damage location can be attained in a very efficient manner. The absolute changes in the curvature mode shapes of the damaged and intact structure are localized in the region of damage and hence can be used to detect the damage in a structure. Curvature damage factors are also calculated with the help of curvature mode shapes for the number of modes considered to detect the damage in a structure.**

Keywords—**Damage Detection, FEM, Mode Shape, Flexibility Matrix.**

I. Introduction

The main objective of damage identification is detection, localization, quantification, classification and prediction of damage. The presence of crack or a localized damage in a structure reduces the stiffness and increases the damping in the structure. From vibration theory, reduction in stiffness is associated with decrease in natural frequencies and modification of the modes of vibration of the structure. Vibration technique has been recognized as an important non-destructive tool for the identification of damage in structures for several years and the technique is undergoing continuous improvement in analysis and instrumentation over the years. The vibration characteristics of a structure can be represented in terms of their modal parameters or the structural parameters. Modal parameters include the frequencies, the modal damping values and the mode shapes associated with each frequency. Structural parameters are the mass, stiffness, flexibility and damping matrices of the structure. Changes in eigen frequency cannot indicate damage at symmetrical locations in a symmetric structure. Therefore, study of mode shape should be introduced for identification of damage location.

The changes in displacement mode shapes are not localized in the region of damage and hence, they do not give indication of the location of damage very precisely. With curvature mode shape in conjunction to natural frequencies, the damage location can be attained in a very efficient manner. The absolute changes in the curvature mode shapes of the damaged and intact structure are localized in the region of damage and hence can be used to detect the damage in a structure.

II. Methodology

In this study, a prismatic beam model has been extended to account for the effect of open transverse cracks. A crack on a beam element introduces considerable local flexibility due to the strain energy concentration in the vicinity of the crack tip under load. A local flexibility can be expressed by a way of a local flexibility matrix, the dimension of which depends on the number of degrees of freedom considered. The analytical method used is based on available expressions for the stress intensity factor (SIF) and the associated expressions for the strain energy release rate (SERR). The adopted approach is similar to the flexibility based matrix method as developed by Papadopoulous and Dimargonas (1987) for studying the influence of an open crack on natural frequencies and mode shapes.

III. Modal Curvature

If a crack or other damage exists in a structure, it reduces the flexural stiffness of the structure at the cracked section or in the damaged region, which increases the magnitude of curvature at that section of the structure. The changes in the curvature are local in nature and hence can be used to detect and locate a crack or damage in the structure. The change in curvature increases with reduction in the value of flexural stiffness and, therefore, the amount of damage can be obtained from the magnitude of change in curvature. From the displacement mode shapes, obtained from the finite element analysis, curvature mode shapes can be obtained numerically by using a central finite difference approximation as

$$u^{//} = \left(u_{n+1} - 2u_n + u_{n-1}\right) / \delta x^2 \qquad (1)$$

where u_n is the vertical component of the displacement mode shapes at node *n* and δx is the length of the element.

The absolute differences in the curvature mode shapes between the intact and damaged beam calculated for number of modes considered give proper indication of the crack. However with this method, different modes may give different predictions. To avoid this difficulty, we need to summarize the results for all the modes, which give proper indication about the crack.

To summarize the results for all the modes, the curvature damage factor as proposed by Wahab and Roeck (1999) is calculated, which can be written as

$$\mathrm{CDF} = \frac{1}{N}\sum_{i=1}^{N}\left|u_{0i}^{//} - u_{di}^{//}\right| \qquad (2)$$

where N is the total number of modes to be considered, $u_0^{//}$ is the curvature mode shape of the intact structure and $u_d^{//}$ is that of damaged structure.

IV. Results and Discussions

The prismatic beams of uniform cross section, simply supported at both ends have been considered for the analysis. Size of the beam is considered as 0.70 *m* x 0.15 *m* x 0.15 *m*. Young's modulus and mass density of the beam material is taken to be 2.58 E10 N/m^2 and 2348.65 kg/m^3 respectively. Open cracks are assumed to be present at different locations of the beam. Crack depths are varied from 2 *cm* to 7 *cm*. Modal parameters such as mode shape are determined using FEM model and from which, absolute differences in the modal curvature between the intact and damaged beam and curvature damage factors are calculated with the help of curvature mode shapes for the number of modes considered.

A. Modal Curvature

For each location of the crack, having different crack depths, absolute differences in the modal curvature (ADMC) between the intact and damaged beam are calculated for the number of modes considered. There is some reduction in magnitude of the curvatures mode shapes due to damping effect.

From the above results, it can be observed that the peaks of the curvature mode shapes are increasing with the increasing crack depth. Fig. 1, 2, 3 and 4 represents the absolute difference of curvature mode shapes of the beam having crack at mid span for crack depths 7 cm, 5 cm, 3cm and 2 cm respectively. At this location of the crack, the peaks of the curvature mode shapes are increasing with the increase in crack depth. These peaks correspond to the location of the crack.

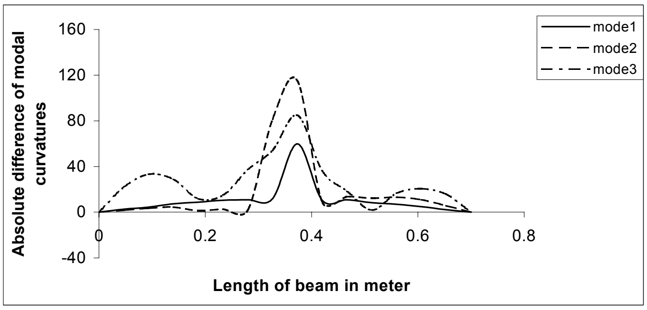

Fig.1 ADMC between cracked and un-cracked beam having crack of 7 *cm* depth at 0.5L

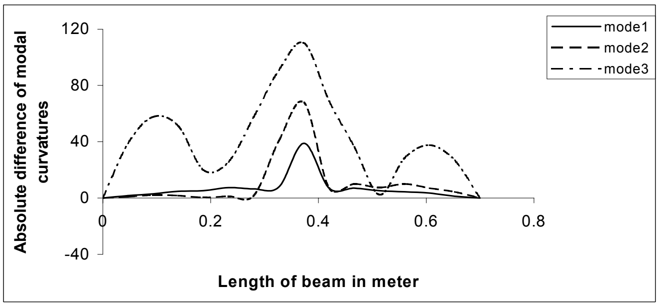

Fig.2 ADMC between cracked and un-cracked beam having crack of 5 *cm* depth at 0.5L

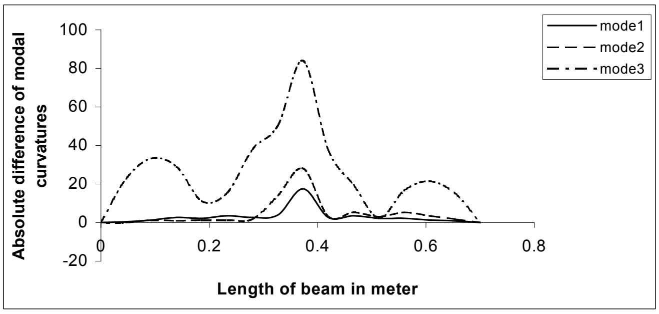

Fig.3 ADMC between cracked and un-cracked beam having crack of 3 *cm* depth at 0.5L

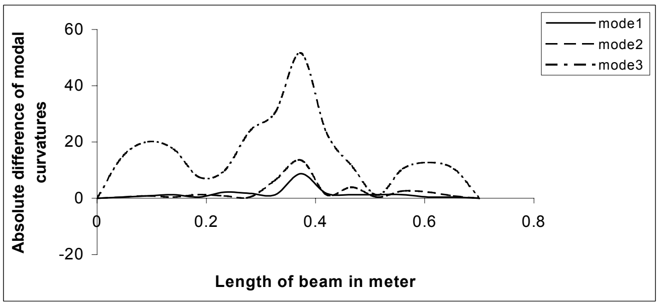

Fig.4 ADMC between cracked and un-cracked beam having crack of 2 *cm* depth at 0.5L

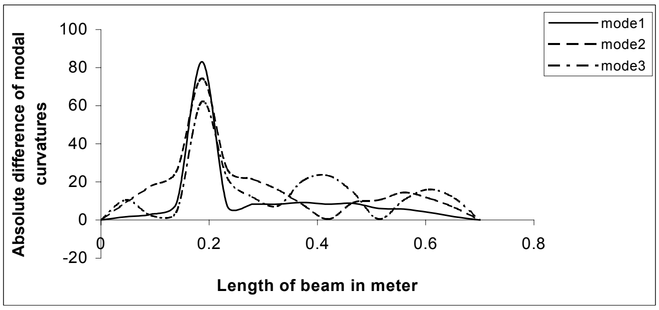

Fig.5 ADMC between cracked and un-cracked beam having crack of 7 *cm* depth at 0.25L

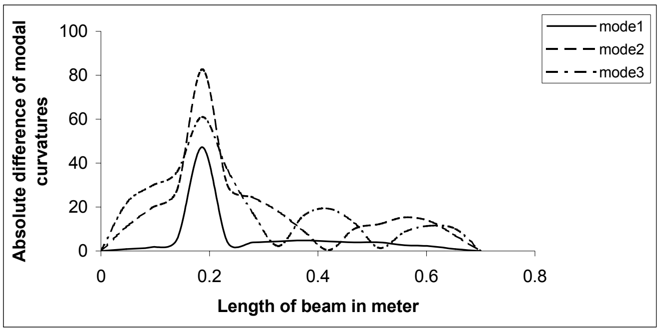

Fig.6 ADMC between cracked and un-cracked beam having crack of 5 *cm* depth at 0.25L

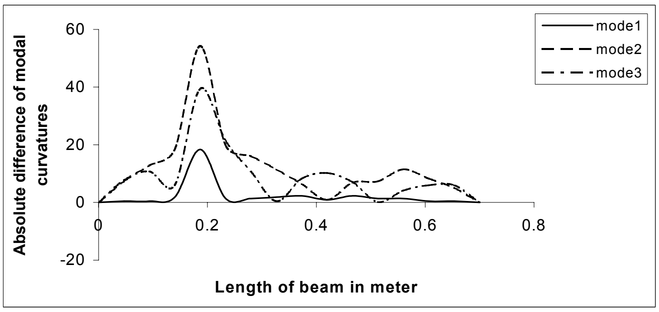

Fig.7 ADMC between cracked and un-cracked beam having crack of 3 *cm* depth at 0.25L

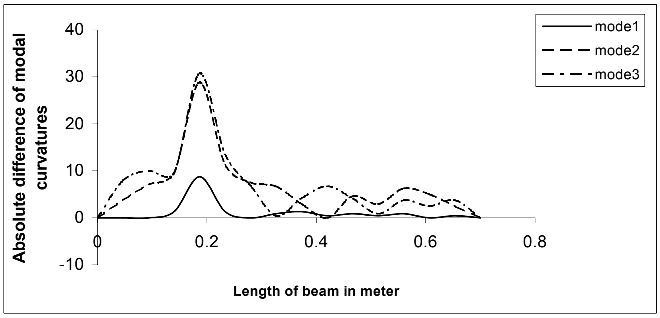

Fig.8 ADMC between cracked and un-cracked beam having crack of 2 *cm* depth at 0.25L

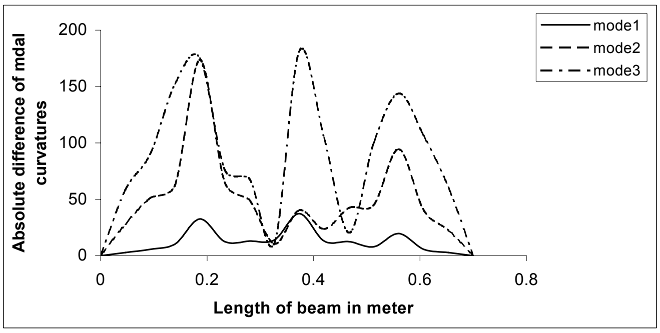

Fig.9 ADMC between cracked and un-cracked beam having multiple cracks of 7 *cm* depth at 0.25L, 0.5L and 0.75L

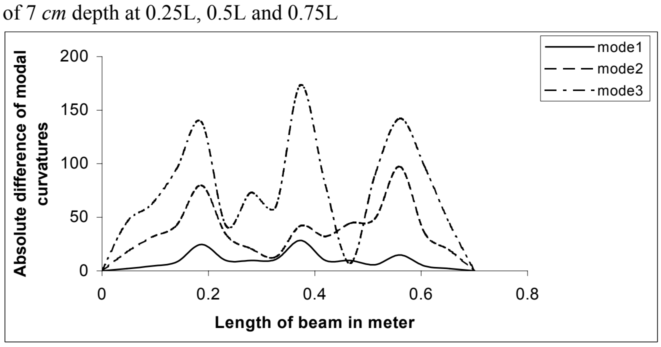

Fig.10 ADMC between cracked and un-cracked beam having multiple cracks of 5 *cm* depth at 0.25L, 0.5L and 0.75L

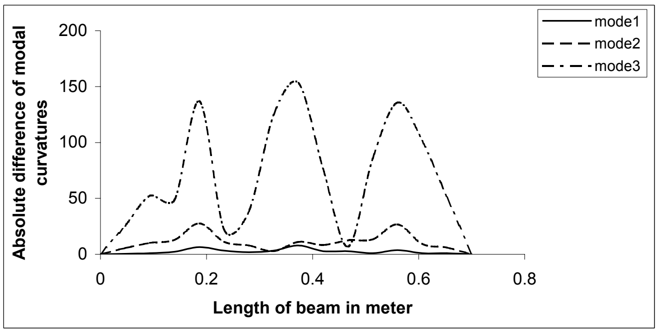

Fig.11 ADMC between cracked and un-cracked beam having multiple cracks of 3 *cm* depth at 0.25L, 0.5L and 0.75L

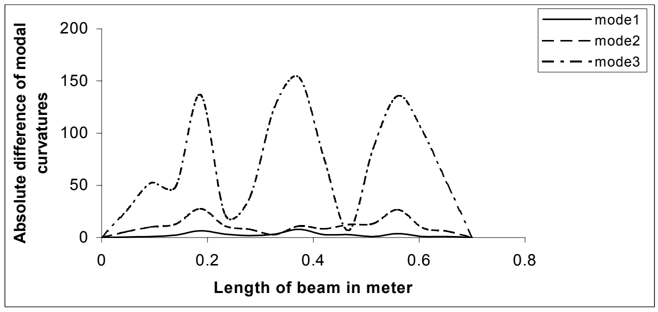

Fig.12 ADMC between cracked and un-cracked beam having multiple cracks of 2 *cm* depth at 0.25L, 0.5L and 0.75L

Fig. 5, 6, 7 and 8 represents the absolute difference of curvature mode shapes of the beam having crack at quarter span for crack depths 7 *cm*, 5 *cm*, 3 *cm* and 2 *cm* respectively. At this location of the crack, the peaks of the curvature mode shapes are increasing with the increase in crack depth. These peaks correspond to the location of the crack.

Fig. 9, 10, 11 and 12 represents the absolute difference of curvature mode shapes of the beam having multiple cracks at 0.25L, 0.5L and 0.75L of the beam for crack depths 7 cm, 5 cm, 3cm and 2 cm respectively. At these locations of the cracks, the peaks of the curvature mode shapes are not always increasing with the increase in crack depth, especially in third mode. This is because of the fact that the calculation of accurate modal parameters for multiple cracks, particularly for higher modes, evaluation of eigen-value problem should be based on some error measure. These peaks correspond to the location of the crack.

B. Curvature Damage Factor (CDF)

For each location of the crack, having different crack depths, curvature damage factors (CDF) are calculated with the help of curvature mode shapes for the number of modes considered. Peak of these CDF indicates the presence of crack. Peak is distinct for higher crack depths. With the help of these, the location of the crack can be determined very precisely.

From the above results, it can be observed that the peaks of CDF are increasing with the increasing crack depth. Fig. 13 represents CDF of the beam having crack at mid span for crack depths 7 *cm*, 5 *cm*, 3*cm* and 2 *cm* respectively. At this location of the crack, the peaks of the curvature mode shapes are increasing with the increase in crack depth. These peaks correspond to the location of the crack.

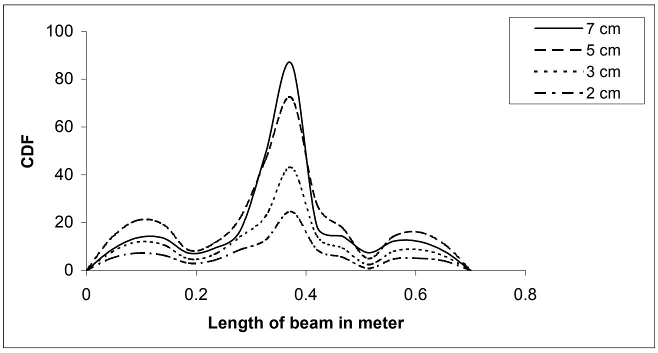

Fig.13 CDF for beam having different crack depths at 0.5L

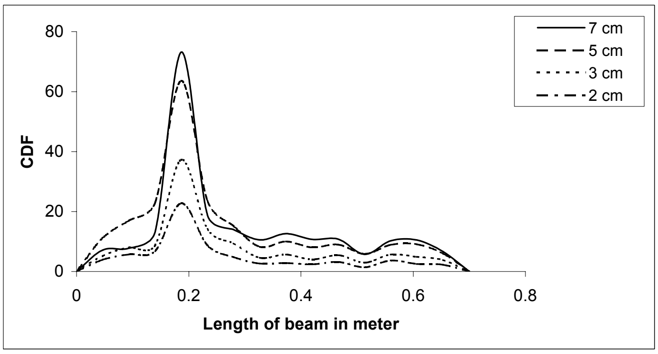

Fig.14 CDF for beam having different crack depths at 0.25L

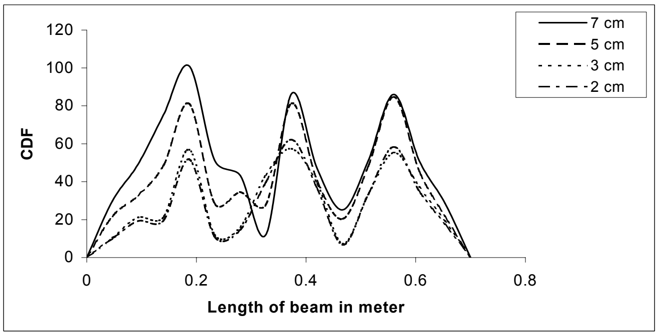

Fig.15 CDF for beam having different crack depths at 0.25L, 0.5L and 0.75L

Fig. 14 represents the CDF of the beam having crack at quarter span for crack depths 7 *cm*, 5 *cm*, 3 *cm* and 2 *cm* respectively. At this location of the crack, the peaks of the CDF are increasing with the increase in crack depth.

Fig. 15 represents the CDF of the beam having multiple cracks at 0.25L, 0.5L and 0.75L of the beam for crack depths 7 *cm*, 5 *cm*, 3 *cm* and 2 *cm* respectively. At these locations of the cracks, the peaks of the CDF are not always increasing with the increase in crack depth. This is because of the fact that the calculation of accurate modal parameters for multiple cracks, particularly for higher modes, evaluation of eigen-value problem should be based on some error measure.

V. Conclusions

In the present case, a study has been carried out for the evaluation of changes in modal parameters of a structure when any degradation takes place. An approach based on the flexibility matrix has been used in conjunction with finite element method to study the influence of an open crack on displacement mode shapes. The analysis has been done for damped model only. The displacement mode shape changes observed in cracked beams was prominent for higher crack depths, whereas, in presence of shallow cracks, displacement mode shape may not be a reliable parameter for damage detection. The changes in displacement mode shapes are not localized in the region of damage and hence, they do not give indication of the location of damage very precisely. With curvature mode shape, the damage location can be attained in a very efficient manner. The absolute changes in the curvature mode shapes of the damaged and intact structure are localized in the region of damage and hence can be used to detect the damage in a structure. Curvature damage factors are also calculated with the help of curvature mode shapes for the number of modes considered to detect the damage in a structure.

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