The Mixed MEWMA and MCUSUM Control Chart Design of Efficiency Series Data of Production Quality Process Monitoring

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Abstract— **A control chart is a crucial statistical tool for tracking the average quality of the dispersion. A more sensitive control chart is also developed to detect minor changes in the efficiency monitoring process, along with the times when using multivariate and mixed models. The well-known multivariate control chart was introduced as T^2 Hotelling; then, to achieve better sensitivity in multivariable, a control chart design was developed for MEWMA and MCUSUM. To find a more sensitive multivariate control chart, it is proposed the control chart MCUSUM type I (MC I) and MCUSUM type II (MC II), and their combination of efficiency as the Mixed MEWMA-MCUSUM type I (MEC I), and the Mixed MEWMA-MCUSUM type II (MEC II). This study was carried out to assess which multivariate control chart is more sensitive by focusing on the ability of the control chart to detect more out-of-control observations in a single control phase. This study used data on the manufacture of wheat flour with 1,380 observations, 30 subgroups, and 46 observations per subgroup. Moisture, ash, and gluten are the quality-related manufacturing data variables used. This study aims to develop the best-mixed control chart design of efficiency for production and quality process monitoring of flour production. Based on the study's findings, the MEC I control chart was shown to be the most sensitive, and this study also demonstrates that it is more sensitive than other multivariate control charts.**

Keywords— **MEWMA; MCUSUM; mixed control chart; efficiency; quality process monitoring.**

I. INTRODUCTION

Wheat flour is a commodity that has an increasingly strategic role even though it is not the staple food of the Indonesian people. The development of national wheat flour needs has changed the role of various government policies to affect the development of the wheat flour industry itself. The domestic sector has invested in substantial amounts, primarily to manage fluctuations in the domestic price of wheat flour from year to year. Consumption of wheat flour continues to increase in Indonesia in line with the growing consumption of instant noodles, bread, biscuits, and cookies. This phenomenon has resulted in wheat flour becoming a basic need with raw materials that must be imported from the international market at high prices. Therefore, the domestic industry is critical for wheat flour production and trade sustainability in Indonesia [1].

The product's composition by standard specifications dramatically affects the quality of the products produced in every industry, including wheat flour. Products that fall outside the specification limits result in poor product results. Statistical Science, which functions to maintain calculations in production activities, manifests the statistical quality control function. The control chart is one of the essential methods in statistical quality control [2]-[4]. The control chart is a process monitoring technique that is widely used in the production process. The control chart monitors and offers valuable data for enhancing operations. Every control chart has a Center Line (CL), Upper Control Limit (UCL), and Lower Control Limit (LCL). A process is considered in control if the data plot falls within the control limits; otherwise, it is said to be out of control. The control chart consists of a univariate control chart that monitors one quality variable, while a multivariate control chart functions to watch more than one quality variable [4].

The well-known univariate control chart has been developed over decades, and the EWMA has been used to monitor high dimensional heteroscedastic processes and compare them to multivariate EWMA [5]-[6]. The EWMA is built by considering exponential weighted as infinitely divisible exponential and gamma distribution [7]-[8]. Besides, there is a CUSUM model for monitoring the coefficient of variation in the textile industry [9]. This univariate CUSUM model is also applied to monitor Poisson count data [10] and also for health and surgical performance [11]-[12]. The earlier multivariate model was introduced using multivariate Hotelling's T^2 control chart. The model has been applied based on kernel density [13] and monitoring multi-dimensional ratios of process means [14]. This control chart was developed to achieve better multi-variable sensitivity into the MEWMA and MCUSUM control charts [15]. The MEWMA control chart has better performance in the mean process [16]-[17], and its modeling for the spread of COVID-19 [18]-[19]. The multivariate EWMA model has significant results on industrial monitoring quality control [15], [20]-[23]. Meanwhile, the multivariate CUSUM also has better performance in bootstrapping [24] and the process mean monitoring and its comparison [25]-[27].

Along with the development of univariate and multivariate control charts, new control charts are more sensitive by comparing the two existing control charts. The combination of univariate quality variables, namely the EWMA and CUSUM control charts, is called the mixed EMWA and CUSUM control chart and its applications [28]-[30]. The research on the mixed multivariate model of MEWMA and MCUSUM control chart has been carried out in an application for wind turbine field [31], and monitoring improvement based on multivariate auto-correlated by residual [32] and variance covariance-matrix [33].

The multivariate control chart is also defined as MCUSUM type I (MC I) and MCUSUM type II (MC II). The combination is referred to as the mixed MEWMA-MCUSUM type I control chart (MEC I), and the mixed MEWMA-MCUSUM type II (MEC II). Meanwhile, the research on the mixed model of MEWMA and MCUSUM control chart has been carried out by monitoring process average, process covariance variance matrix, and multivariate auto-correlated process control. This study demonstrates that by creating additional out-of-control data plots, the mixed model control charts are very sensitive to identifying small process shifts.

Based on the availability of data, this research was carried out using a time-weighted control chart using the mixed multivariate design model. Meanwhile, it proposed the multivariate control chart design MEWMA, MCUSUM, MC I, MC II, MEC I, and MEC II. To choose the most effective and comparable control chart from the available wheat flour production data, the quality control chart is created by tracking the process average of mixed model.

II. MATERIAL AND METHOD

This research is conducted to develop an efficient combination of multivariate control chart designs for monitoring quality control of flour production. The 1,380 observations that comprise the data series observation of flour production are divided into 30 subgroups, each with 46 observations. The method uses statistical modeling of quality

control of multivariable and their mixture, as in the following subsection.

A. The Control Chart Design of MEWMA and MCUSUM

The multivariate control chart is designed into several models by referring to MEWMA and MCUSUM in this section [34]. The MEWMA control chart is a generalization of the EWMA univariate diagram. It is known that the MEWMA statistical value depends on the MEWMA vector value with the same formula form as the EWMA value. So that the statistical value of the MEWMA control chart is given by:

$$T_i^2 = Z_i \Sigma_{Z_i}^{-1} Z_i, \tag{1}$$

where $L > 0$ is selected to reach ARL in control, and ΣZ_i is the variance-covariance matrix of Z_i. The exact weight is assigned to each quality variable where $\lambda = \lambda_j$ for $j = 1, 2, ..., p$. The MEWMA vector can be written as follows:

$$Z_j = \lambda X_i + (1 - \lambda) Z_{i-1} \tag{2}$$

where Z_i is the MEWMA vector and λ is the MEMWA weight for $0 < \lambda \leq 1$.

Based on the weighted values, the covariance variance matrix of Z_i can be written in the form of weights (λ), and the covariance variance matrix of Z_i which ΣZ_i denotes, for $i = 1, 2, ..., n$ is:

$$\Sigma_{Zi} = \left(\frac{\lambda}{2-\lambda}\right) \Sigma \square \tag{3}$$

and

$$\Sigma = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})' \tag{4}$$

where $\bar{X} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_p)'$ and $\bar{x}_j = (1/m) \sum_{i=1}^{m} x_{ij}$.

The upper limit of the MEWMA control chart or Upper Control Limit (UCL) is defined as UCL = H where the value of H was influenced by the quality variable (p), weighting, and ARL under controlled conditions (ARL$_0$). Data control can be constructed in advance by using standardized data. The lower limit of the MEWMA control chart or the Lower Control Limit (LCL) is defined as LCL = 0.

From the cumulative sum control chart, the multivariate CUSUM (MCUSUM) control chart is derived. as introduced for the first time by Crosier [34] formerly called the Crosier MCUSUM control chart (CMCUSUM), which monitors the average of the normal multivariate process. Crosier MCUSUM control chart defines statistics C_i from MCUSUM after observing the average of the i-th vector as follows:

$$C_i = ((S_{i-1} + X_i)' \Sigma_{\square}^{-1} (S_{i-1} + X_i))^{1/2}, \tag{5}$$

where $S_i = 0$ for $C_i \leq k$, $S_i = (S_{i-1} + X_i)(1 - k/C_i)$ for $C_i > k$, and $S_0 = 0$ for $k > 0$.

This CUSUM multivariate scheme denoted by MCUSUM has an UCL called H^*, affected by shift (δ), ARL, and the number of quality variables p, where the number of shifts is as follows:

$$\delta = (\mu' \Sigma_0^{-1} \mu)^{1/2}. \tag{6}$$

where $H^* < U_i$, then the process is out of control, where the value of U_i is defined as,

$$U_i = (S_i \Sigma_0^{-1} S_i)^{1/2}. \tag{7}$$

The next development of MCUSUM control chart is influenced by the root of the distance of each sample mean μ_0 and accumulates the average squared distance of the i-th observations from the target value μ_0 [15]. The first type of model is called the control chart MCUSUM type I (MC I), this is starting by introducing the variables $X_{i1}, X_{i2}, \ldots, X_{ip}$ where i is observing i and p quality variables, for $i=1,2,\ldots,n$. The variable X_{ip} can be in the form of individual or subgroup observations assuming a normal distribution where $X_{ip} \sim N(\mu, \Sigma)$, for μ and Σ is the covariance variance matrix of size $p \times p$. The MC I's statistical values [13],[15] are defined as follows:

$$C_i = \sum_{j=i-n_i+1}^{i}(X_j - \mu_0) \qquad (8)$$

where n_i is the number of subgroups. Therefore, the value of $(1/n_i)C_i$ can be written as follows:

$$(1/n_i)C_i = \left((1/n_i)\sum_{j=i-n_i+1}^{i}X_i\right) - \mu_0 \qquad (9)$$

The difference between the target value and the mean, as well as the cumulative sample mean, is represented by the vector $(1/n_i)C_i$. Consequently, estimator $(1/n_i)C_i + \mu_0$ may be used to estimate the multivariate mean at process time i. The norm of C_i can be written as follows:

$$\|C_i\| = \sqrt{C_i' \Sigma^{-1} C_i} \qquad (10)$$

which is viewed as the approximate distance of the process average from the process target average. A multivariate control chart is formed with the definition of MC I as follows:

$$MCI_i = max\{\|C_i\| - kn_i, 0\}, \qquad (11)$$

where $n_i = n_{i-1} + 1$ for $MCI_{i-1} > 0$, and $n_i = 1$ for $MCI_{i-1} \leq 0$. The MC I control chart is operated by plotting the value of MC I_i to the control diagram. The UCL is called H^{**}. If MC $I_i > H^{**}$, then the process is considered out of control.

The second type of the MCUSUM variation model is continued as the control chart MCUSUM type II (MC II), where the MC II's statistical values are defined as follows:

$$D_i^2 = (X_i - \mu_0)\Sigma^{-1}(X_i - \mu_0) \qquad (12)$$

which has distribution χ^2 with significance level α when the in-control process has distribution χ^2 non-central when the production process is out of control. The control chart MC II was formed as follows:

$$MCII_i = max\{0, MC\ II_{i-1} + D_i^2 - k\} \qquad (13)$$

with MC $II_0 = 0$. The control limit is called H^{***}. The condition of MC $II_i > H^{***}$, then the process is considered out of control.

The selection of parameter k for the MC II control chart is different from the other control charts because the target state of MC II is not symmetric. Therefore, the value of k for control chart MC II is filled by $k = p + (1/2)\lambda\mu_1$ where p is the number of quality variables, the parameter λ is the size of weighting, and μ_1 is the specific value on the target state where $\mu_1 = 1$ [28].

B. The Proposed Model of the Mixed MEWMA-MCUSUM Control Chart Design

The proposed model of the mixed MEWMA and MCUSUM Control Chart Design is developed for efficiency detection for quality process monitoring. The first type is called the mixed MEWMA-MCUSUM type I (MEC I) and is developed by following steps where the control chart is obtained from integrating the ordinary MEWMA statistics on MC I. For example, the variables of $X_1, X_2, \ldots, X_n$, untuk $i=1, 2, \ldots, n$ is a sample of n monitored quality variables. The MEC I control chart was developed by transforming the MEWMA statistical sample and integrating it into MCUSUM [34], which is as follows:

$$MEC_i = max\{0, MEC_{i-1} + (Z_i - \mu_0) - k^*\}, \qquad (14)$$

where $MEC_0 = 0$, and

$$k^* = k\frac{(MEC_{i-1} + Z_i - \mu_0)}{((MEC_{i-1} + Z_i - \mu_0)'\Sigma_{Z_i}^{-1}(MEC_{i-1} + Z_i - \mu_0))^{1/2}} \text{ for } k > 0.$$

If the condition $k^* \geq MEC_{i-1} + (Z_i - \mu_0)$ is satisfied then it is clear to obtain $MEC_i = 0$, so that the statistical value for the MEC I diagram can be defined as follows:

$$MEC\ I_i = MEC_i' \Sigma_{Z_i}^{-1} MEC_i \qquad (15)$$

The UCL on the MEC I control chart is called H^{****}. If the condition MEC $I_i > H^{****}$, then the process is said to be out of control.

The mixed control chart type II is called the mixed MEWMA-MCUSUM type II (MEC II) and developed by following steps. The MEC II control chart is the control chart obtained from integrating the ordinary MEWMA statistics on MC II. The usual MEWMA statistic is transformed to a cumulative sum vector of MC II. The variable Z_i was statistically distributed with the mean of μ_0 and the variance-covariance matrix Σ_{Zi} [34] which is defined as follows:

$$\Sigma_{Zi} = (\lambda/2 - \lambda) \qquad (16)$$

while the cumulative sum vector is defined as
$S_i = \sum_{j=i-n_i-1}^{i}(Z_j - \mu_0)$.
Based on the statistics generated from S_i, it is given the following equation:

$$MECII_i = max\{0, (S_i'\Sigma_{Z_i}^{-1}S_i)^{1/2} - k_1 n_i\} \qquad (17)$$

where $k_1 = k((\mu_1 - \mu_0)'\Sigma_{Z_i}^{-1}(\mu_1 - \mu_0))^{1/2}$ for $k > 0$, $n_i = n_{i-1} + 1$ for MEC $II_{i-1} > 0$, and $n_i = 1$ for MEC $II_{i-1} \leq 0$. The upper control limit on the MEC II control chart is called H^{*****}. If the condition MEC $II_i > H^{*****}$, then the process is said to be out of control.

III. RESULTS AND DISCUSSION

In this research has developed a new model of the mixed MEWMA and MCUSUM by using multivariate data from wheat flour production in terms of moisture, ash, and gluten. Wheat flour's water content is shown by moisture. Wheat flour's shelf life may be shortened if the moisture content is beyond the maximum allowable level since it will decay more quickly and develop mold and a musty odor. The moisture specification limit for wheat flour is 13.5%-13.9%. Ash in the flour has an impact on the production and end results, including product color and dough stability. The lower the ash content, the better the flour will be. Ash content that is more than 0.57% indicates that the product has failed. Gluten is a chewy, elastic substance found in wheat flour that influences the nutritional value of the food it makes. The amount of

protein in the flour increases as the gluten concentration does. The specification limit for gluten in wheat flour is 22% -26%.

The collected data of variables moisture, ash, and gluten are presented in Table 1, and the mean of each variable satisfied the limit of specification, even though some of data out of control based on their minimum and maximum values. This shows the importance of observing the data to detect whether the data is out of the control limits in monitoring the production process. Detection of data out of control will be analyzed through a multivariate control chart.

TABLE I
STATISTICAL DESCRIPTIVE OF FLOUR QUALITY

Variable	Limit of Specification	Mean	Variation Coefficient	Min	Max
Moisture	13.5 - 13.9	13.35738	0.02965	12.59	14.35
Ash	≤ 0.57	0.53780	0.00934	0.44	0.67
Gluten	22 - 26	23.31980	0.12764	18.93	29.65

The analysis of a multivariate control chart requires multivariate normal data. Based on Figure 1, it can be seen that the q-q plot follows a linear pattern, which is expected to be multivariate normal data. The evidence is done by examining the Pearson correlation coefficient by using Mahalanobis distance and q_i as the chi-square value for each observation. If the value of the Pearson correlation coefficient is greater than the table percent point value of the normal probability plot correlation coefficient, then the initial hypothesis will be rejected, which means that the data is not normally distributed multivariate, and vice versa.

Based on Table 2, the Pearson correlation value is 0.969 when it is compared with the table value of the percent point of the average probability plot correlation coefficient for a significance level of $\alpha = 0.05$ and $n = 46$, with a table value of 0.974 obtained. This means that the correlation value is smaller than the table value, so the initial hypothesis is not rejected, which means that the data is multivariate normal. Since the data is multivariate normal, then suppose the value of the Pearson correlation coefficient is greater than the table percent point value of the normal probability plot correlation coefficient. In that case, analysis of the multivariate control chart can be advanced.
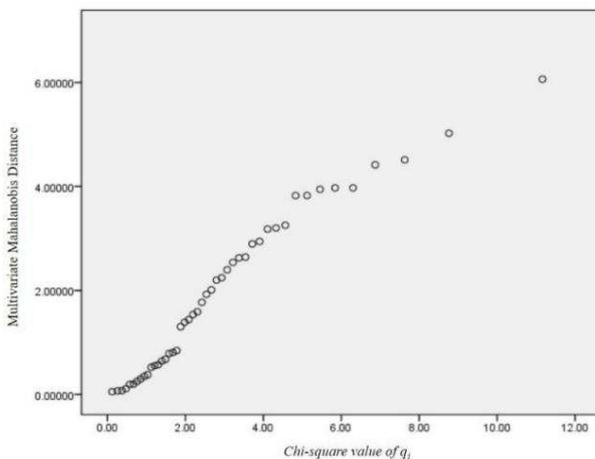

Fig. 1 Graphical normal multivariate test by using q-q plot.

TABLE II
PEARSON CORRELATION OF VARIABLE BY USING MAHALANOBIS DISTANCE

		Mahalanobis Distance	qi
Mahalanobis Distance	Pearson Correlation	1	0.969**
	Sig. (2-tailed)		0.000
	N	46	46
qi	Pearson Correlation	0.969**	1
	Sig. (2-tailed)	0.000	
	N	46	46

In this research, the multivariate control chart MEWMA, MCUSUM, MC I, MC II, MEC I, and MEC II were applied to wheat flour production data with three variables. These variables are moisture (X_1), ash (X_2), and gluten (X_3). The following is a comparison of the results of the application of multivariate control charts using MEWMA, MCUSUM, MC I, MC II, MEC I, and MEC II as in Figure 2.

Based on Figure 2 (a), the MEWMA control chart uses a weight $\lambda = 0.25$ and $ARL_0 = 200$ detects that there is 1 out of control observation that is equal to 2.17%. Out of control observation data is found in the 14th observation according to the T_i^2 value using Equation (1). When viewed from the data plots in the observations, the resulting pattern in this uncontrolled control chart is an irregular model. This indicates a special cause for changes in the production process, with plot data fluctuating even though it is still within the control limits. Meanwhile, based on Figure 2 (b), the MCUSUM control chart detects that there are no out-of-control observations examined based on Equation (7). Even though there are no out-of-control observations on the control chart, the data plot of the MCUSUM control chart found that there are seven data that all increase in a row, namely at the 8th to 14th observation. According to the out-of-control observation criteria, this shows that although the data plot is below the control limit, the resulting data is still uncontrolled.

Based on Figure 2 (c), the MC I control chart using the parameter value $k = 0.5$ detects that there are 5 out of control observations, namely 10.87%. Out of control observational data are found in the 14th, 15th, 16th, 17th and 18th observations with the MC Ii value determined based on Equation (11). When viewed from the data plots in the observations, the pattern produced in this uncontrolled control chart is a mixed model. This indicates that the company uses raw materials from several suppliers, causing the process to get out-of-control. Meanwhile, based on Figure 2 (d), the MC II control chart detects that there are not out of control observations, which shows that all observations are under control.

Based on Figure 2 (e), the MEC I control chart using the parameter value $k = 0.5$ detects that there are 29 out of control observations, namely 63.04% with the MEC Ii value determined based on Equation (15). Meanwhile, based on Figure 2 (f), the MEC II control chart detects that there are 20 out of control observations with MEC IIi values determined based on Equation (17). When viewed from the data plots in the observations, the patterns produced on the MEC I and MEC II control charts are out of control, which change suddenly and gradually. This indicates that there is a special reason for changes in the production process, namely when the company uses raw materials from several suppliers, causing the process to become uncontrolled.
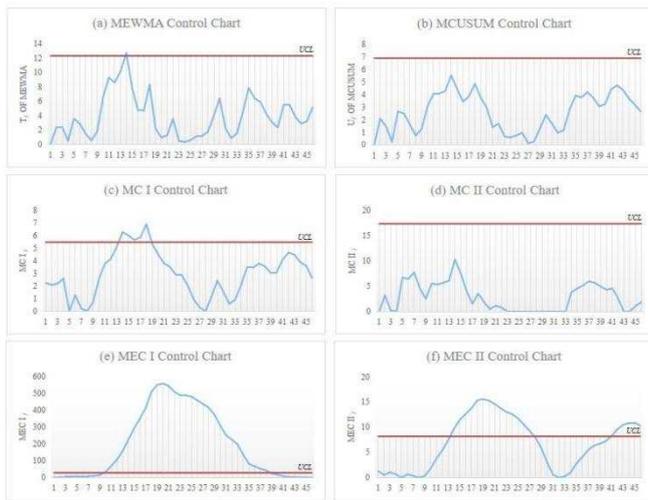
Fig. 2 Application of Multivariate Control Diagrams on control charts diagram (a) MEWMA, (b) MCUSUM, (c) MC I, (d) MC II, (e) MEC I, and (f) MEC II.

After creating control diagrams for MEWMA, MCUSUM, MC I, MC II, MEC I, and MEC II, the next thing to do is compare the results of the six control charts. The comparison was conducted to determine which control chart is more effective in detecting out-of-control observations. The number of observations that are out of control shows how the control chart compares. The control chart will be more sensitive the more out-of-control observations it can identify. The multivariate control chart MCUSUM and MC I do not detect data out-of-control. These two multivariate control charts belong to low sensitivity to detect data out-of-control. While controlling the MEWMA and MC charts, I only detected a few numbers of data that were out-of-control. The mixed multivariate control chart MEC II is relatively better at detecting data out-of-control with faster and more gradual changes. Furthermore, the mixed control chart MEC I give the best detection data out-of-control. The more detailed data out-of-control for each multivariate control chart is presented in Table 3.

TABLE III
COMPARISON OF OUT-OF-CONTROL OBSERVATIONS ON MULTIVARIATE
CONTROL CHARTS OF MEWMA, MCUSUM, MC I, MC II, MEC I AND MEC II.

Control Chart	Total Observations Out of Control	Control Chart Pattern
MEWMA	1	Irregular
MCUSUM	0	-
MC I	5	Mix
MC II	0	-
MEC I	29	Fast and gradual changes
MEC II	20	Fast and gradual changes

Table 3 shows that all observations on the MCUSUM and MC II control charts do not have out-of-control observations. However, on the MCUSUM control diagram, when identified against the observation data plot in Figure 1, a data plot pattern identifies that the observation is out of control. As for the MEWMA control diagram, MC I, MEC I, MEC II have out-of-control observations. However, the MEC I control chart has the highest total out-total control observations. Thus, the MEC I control chart can be more sensitive than the other

control charts. The quality variable in the controlled multivariate observation is a variable with a multivariate normal distribution. Therefore, the MEC I control chart was chosen as the most efficient control chart for controlling multivariate data compared to other multivariate control charts. The difference in the pattern produced on the control diagram causes confirmation to the company regarding the state of the production process that occurs so that the cause of the runaway process can be adequately identified.

IV. CONCLUSION

Multivariate control charts come in a variety of forms, and they are used to assess sensitivity by identifying data that is out of control in one control phase. In order to develop a more sensitive control chart, a new model of the mixed MEWMA and MCUSUM was introduced by using multivariate data from wheat flour production in terms of moisture, ash, and gluten. Based on the results of multivariate control chart models and their combinations with the flour production data, it is obtained that the mixed MEC I control chart is the most sensitive control chart. The MEC I control chart is more efficient because it can detect the total number of out-of-control observations in only one phase of control. This control chart design is very efficient in monitoring the production quality process for wheat production by using three indicators of the quality of the wheat flour.

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