Mechanistic Modeling of Conventional Rail Tracks to Predict the Total Permanent Deformation

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Abstract—A traditional ballasted railway track, characterized by its layers of ballast, sub-ballast, and subgrade, is the most viable option for Indonesia's railway system, primarily due to its cost-effectiveness and straightforward construction process. This track type must ensure a stable train route, maintaining appropriate horizontal and vertical alignment. Each component of the system is required to fulfill its designated role effectively. However, prior research has predominantly concentrated on assessing the permanent deformation of individual layers within conventional rail tracks. It has been determined that employing a linear elastic material model is inadequate for accurately representing the behavior of ballast, sub-ballast, and subgrade. The prevailing approach in existing literature involves simulating the inelastic behavior and modeling the permanent deformation of granular and soil materials in railway tracks using elastoplastic constitutive models, such as the Mohr-Coulomb and Drucker-Prager (Elastic Perfect-Plastic) models. In this context, the present study aims to evaluate the efficacy of mechanistic modeling in predicting the overall permanent deformation of conventional rail tracks. A key finding from this investigation is that the ballast layer to the total permanent deformation in the design of conventional rail tracks. A key finding from this investigation is that the ballast layer plays a crucial role in the permanent deformation of the conventional track, followed by the subgrade and sub-ballast layers.

Keywords— Conventional rail track; finite element method; mechanistic modeling; modified drucker-prager cap; permanent deformation.

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I. INTRODUCTION

Historically, Indonesia's railway infrastructure has relied exclusively on traditional ballasted track systems, which utilize ballast, sub-ballast, and subgrade layers to ensure structural stability. This choice has been primarily driven by budget constraints and the simplicity of construction associated with such systems [1]. In 2022, the length of conventional ballasted track railway lines in Indonesian railway systems is 6,221.70 kilometers. Unfortunately, Indonesia's traditional rail track deterioration, poor performance, and rutting were the primary causes of the majority of train derailment accidents on railroads (see Figure 1).

The investigation revealed that employing a linear elastic material model proved inadequate for accurately simulating the behavior of ballast, sub-ballast, and subgrade materials [2-4]. The general practice, as shown in the literature, is to

simulate the inelastic behavior and to model permanent deformation of granular and soil materials in railway tracks through elastoplastic constitutive relationships, such as the Mohr-Coulomb Model [5-8] and Drucker-Prager (Elastic Perfect-Plastic) Model [9-11], as shown in Figure 2.



Fig. 1 Train derailment accidents due to conventional rail track deterioration and poor performance

Cohesion plays a critical role in determining the tensile strength of materials as described by the Mohr-Coulomb model. It represents the component of shear strength in granular materials that remains independent of interparticle friction. The Mohr-Coulomb model encompasses both elastic and fully plastic behavior, incorporating associated and nonassociated flow laws. The yield surface defined by this model takes the form of a hexagonal pyramid that extends into the three-dimensional space of compressive principal stresses. Nevertheless, the six-faced yield surface presents mathematical difficulties along each edge, as the normal vector of the yield surface cannot be distinctly identified along these edge lines.



Fig. 2 a. Mohr Coulomb Model and; b. Drucker-Prager (Elastic Perfect-Plastic) Model

The Drucker-Prager (D-P) model is favored due to its straightforward approach to representing gravel's elasticplastic behavior. It employs a rounded, cone-shaped yield surface, contrasting with the abrupt and complex yield surface associated with the Mohr-Coulomb failure criteria in stress space. As a result, the D-P model was developed to overcome numerical computation issues. Progressive yielding does not affect the yield surface; hence, the material is elastic-perfectly plastic, and there is no hardening rule.

The behavior of ballast is significantly influenced by pressure. Given that the D-P model operates as an elastic perfect-plastic framework, the outcomes of the simulations were not entirely satisfactory. Consequently, the Modified Drucker-Prager model with Cap Hardening (MDPC), illustrated in Figure 3, can be used to represent better frictional materials, which are generally granular in nature, such as soils and rocks. These materials exhibit yield strength that is contingent upon pressure, meaning that their stiffness and strength increase with rising pressure or stress.

The MDPC model is superior to both the Mohr-Coulomb and D-P models for capturing the mechanical performance of railroad aggregate materials. In other words, isotropic hardening is considered a substitute for perfect plasticity to strengthen the correlation with the measurements [12], [13].



Fig. 3 The Modified Drucker-Prager Model with Cap Hardening

The primary role of the unbound granular layer is to withstand lateral, vertical, and longitudinal loads exerted by trains, distributing these forces from the sleepers to the underlying subgrade. Furthermore, the subgrade is an integral part of track construction, and its qualities are crucial to track performance and track quality. Due to variations in material qualities, the degree of PD of each structural layer will vary. Ballast and sub-ballast on railroad tracks deform mostly due to frictional slip and particle breakage [14], [15]. Applying cyclic loading to ballast and sub-ballast materials results in a progressive buildup of plastic deformation, a decrease in voids, and an enhancement in stiffness [16]. It has been established that settlement within the track foundation is a primary contributor to track irregularities [17], [18]. Traditional track systems are prone to settlement due to repetitive dynamic loads and various geotechnical factors, including consolidated settlements and insufficient soil subgrade compaction [7].

The track must provide a stable path for the train with proper horizontal and vertical alignment. Each system component must perform its specific functions satisfactorily. Unfortunately, previous studies mainly focused only on capturing the permanent deformation of a single or a particular layer (ballast, sub-ballast, or subgrade) in conventional ballasted rail track [8], [19], [27].

A comprehensive understanding of the total permanent deformation in conventional rail tracks could facilitate the development of more economically viable rail track systems that necessitate reduced maintenance efforts. However, before achieving this goal, it is essential to gain insights into the permanent deformation contributions of each layer within the conventional rail track structure, including ballast, sub-ballast, and subgrade layers. Furthermore, these contributions must be systematically compared and subsequently correlated with the overall permanent deformation of the rail track. This study explores the efficacy of finite element modeling in predicting the total permanent deformation of conventional rail tracks, while also assessing the individual contributions of the various component layers to this total deformation, utilizing Abaqus Software for the analysis.

II. MATERIALS AND METHOD

A. Structural Geometry and Dimension

The conventional track with a thickness of 30 cm ballast, 30 cm sub-ballast, and 330 cm of subgrade is depicted in Figure 4.



Fig. 4 Geometric and layer thicknesses of conventional rail tracks for numerical modeling

B. Material Properties

1) Elastic Parameters of Rail Track Materials: The characteristics of the elastic behavior of traditional rail track materials, including Mass Density (ρ), elastic modulus (E), and Poisson's Ratio (ν), are presented in Table 1.

 TABLE I

 Elastic behavior parameter for simulation [7], [9]-[10], [19]

Layer	ρ [kg/m3]	E [MPa]	v
Sleeper	1833.30	30,000	0.20
Ballast	2192.39	150	0.33
Sub-Ballast	1937.46	120	0.30
Subgrade	2192.39	20	0.33

2) Elasto-Plastic Parameters of Ballast, Sub-Ballast, and Soil Subgrade: The elastic characteristics of ballast, subballast, and soil subgrade were analyzed by applying the MDPC model. This model is specifically designed to simulate the behavior of unbound aggregates and soil materials, particularly those that demonstrate pressure-dependent yield and failure mechanisms. In this context, the material's response to stress is characterized by its tendency to yield or fail under varying pressure conditions. The material parameters that correlated with the MDPC model were separated into 3 (three) distinct groups, which are the elasticity parameters (see Table 1), Drucker-Prager-Cap Plasticity Parameters (see Table 2), Cap Hardening Parameters (see Table 2).

TABLE II	
DRUCKER-PRAGER-CAP PLASTICITY AND CAP HARDENING PARAMETERS [7, 9-	10]

Drucker-Prager-Cap Plasticity					Cap Hardening			
Materials	Material Cohesion, d (MPa)	Angle of Friction β	Cap Eccentricity R	Initial Cap Yield Surface Position, Pa (MPa)	Transition Surface Radius, α	Flow Stress Ratio K	γ_1	γ2
Ballast	0.001	45	1.8576	0.5	0	1	0.5	7
Sub-Ballast	0.001	45	2.3201	0.4	0	1	0.4	6
Subgrade	0.012	14.3	5.8405	0.3	0	1	0.3	5

C. Loading Systems

The dynamic load magnitude of the Babaranjang freight train was determined by applying Equation 1 [28] and Equation 2 [24], subsequently converting it into a concentrated force exerted on both the left and right rails as a cyclic load. Figure 5 illustrates the characteristics of a Babaranjang freight wagon, which possesses a maximum payload capacity of 18 tons. The separation between the two bogies measures 10.830 meters. This measurement was used with Equation 3 [29-30] to compute the passing frequency from one bogie to another within the cyclic loading systems.



Fig. 5 Dimension of Freight Wagon in Babaranjang Freight Train Set [31]

$$I_p = 1 + 0.01 \left(\frac{V}{1.609} - 5\right) \tag{1}$$

$$P_d = P_s \times I_p \tag{2}$$

where:

 I_p = Conversion Factor

V = Babaranjang Train Speed, (kph)

 P_s = Static Wheel Load, (kg)

 P_d = Dynamic Wheel Load, (kg)

$$f_p = \frac{V}{d} \tag{3}$$

where: f_p = Passing Frequencies, (Hz)

 $\dot{V} = 40 \text{ kph}$

D = 10.830 m

This research investigated the response of conventional rail tracks under cyclic loading systems. The loading conditions of Babaranjang freight trains, specifically regarding train speed, were utilized to assess their impact on the overall permanent deformation of the railway tracks.



Fig. 6 Mesh of Simulation Models of Conventional Track

D. Finite Element Model Simulation

The 2D numerical modeling, meshing, constraint, and loading were carried out using Abaqus/finite CAE's element software to simulate the conventional rail track subjected to Babaranjang freight train loading systems. The simulation model was sketched in Figure 4, while the 2D mesh is displayed in Figure 6.

III. RESULTS AND DISCUSSION

A. Permanent Deformation of Conventional Track

Figure 7 shows the permanent deformation of each layer in the conventional track with the MDPC model. The data in Figure 7 show that the ballast layer has the highest permanent deformation, followed by the subgrade and sub-ballast layers.



Fig. 7 Permanent deformation of conventional track with MDPC model: a). ballast; b). sub-ballast; c). subgrade; d). combination of all layers

B. Contribution of the Layers in Rail Tracks to the Total Permanent Deformation

The examination of the percentage contribution of each layer to the overall permanent deformation of conventional, AO, and AU tracks is presented in Table 3. In the case of the conventional track, the contributions of the ballast, sub-ballast, and subgrade layers to the total permanent deformation are 71.72%, 13.23%, and 15.05%, respectively. Figure 8 compares the permanent deformation of conventional track at 100,000 and 1 million loading cycles. It is apparent from these figures that the magnitude of permanent deformation of conventional rail track types significantly increases as loading reaches 1 million cycles.

TABLE III
CONTRIBUTION OF THE LAYERS IN CONVENTIONAL RAIL TRACKS TO THE
TOTAL PERMANENT DEFORMATION

Layer	Contribution (%)	mm
Ballast	71.72%	1.2979
Sub-Ballast	13.23%	0.2395
Subgrade	15.05%	0.2723
Total	100%	1.8097



Fig. 8 Permanent Deformation of Conventional Track with MDPC Model at 100,000 and 1,000,000 Loading Cycles: a). Comparison of Each Layers; b). Combination of All Layers

C. Permanent Deformation Prediction and Measurement in Previous Studies

Table 4 summarizes the selected studies and works on predicting and measuring permanent deformation of rail track structures (ballast, sub-ballast, or subgrade layer). Most of the studies were conducted based on laboratory experiments. The magnitude of permanent deformation was obtained due to a particular number of loading cycles and specific axle load. It can be concluded that the magnitude of permanent deformation of ballast, sub-ballast, and subgrade layer predicted by this study is reasonable and within the range of the laboratory experimental results performed by previous research.

TABLE IV
VARIOUS STUDIES ON THE PREDICTION AND MEASUREMENT OF PERMANENT DEFORMATION OF RAIL TRACK STRUCTURES

Paper	Methodology: Field Test or Lab Experiment	Tested Materials	Measured Permanent Deformation (mm)	No. of Loading Cycles (LC)	Results from Current Study (1,000,000 LC, 18 Tons Axle Load)
Grossoni	Southampton Railway	Ballast	20 mm	1,000,000 LC of 32 Tons	5.60 mm of Ballast
et al. [20]	Testing Facility (SRTF)	(Conventional		Axle Load	Permanent Deformation in
		Track)	10 mm	1,000,000 LC of 20 Tons Axle Load	Conventional Track
Zhang	Full-scale model testing	Lime-stabilized	5.8 mm	3,000,000 LC of 18 Tons	4.90 mm of Subgrade
and Jiang	-	weathered red		Axle Load	Permanent Deformation in
[32]		mudstone subgrade with thickness of 2.5 m	7.6 mm	4,000,000 LC of 25 Tons of Axle Load	Conventional Track
Abadi et al. [21] and Abadi et al. [33]	Southampton Railway Testing Facility (SRTF)	Ballast with 300 mm of thickness	10 mm	3,000,000 LC of a 20 tons axle load	5.60 mm of Ballast Permanent Deformation in Conventional Track
Ionescu et al. [34] in Aingaran [23]	Laboratory Test	Ballast	12 mm	1,000,000 LC of 25 Tons Axle Load	5.60 mm of Ballast Permanent Deformation in Conventional Track
Brown et al. [35]	The Nottingham railway test facility, a full-scale laboratory test facility for railway trackbeds.	Ballast	7.5 mm	1,000,000 LC. Application of cyclic loads of up to 94- 100 kN of Axle Load	5.60 mm of Ballast Permanent Deformation in Conventional Track

IV. CONCLUSION

This paper presents computational modeling and simulation of conventional rail tracks considering the Modified Drucker-Prager Cap constitutive relationship of unbound granular and soil layers. The model simulation results of each layer and total permanent deformation were evaluated. The most obvious finding from this study is that the ballast layer contributes most to the total permanent deformation of the conventional track. Further study is suggested to model and compare the total permanent deformation of conventional and other rail tracks.

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CONFLICT OF INTEREST STATEMENT

The corresponding author states that there is no conflict of interest.

AUTHOR CONTRIBUTION

The contributions of the authors to this paper are delineated as follows: the conception and design of the study were undertaken by Dian M. Setiawan and Sri Atmaja P. Rosyidi; Aminudin Syah and Renandri Abdillah Rachman performed data collection; the analysis and interpretation of the results were conducted by Dian M. Setiawan, Sri Atmaja P. Rosyidi, and Aminudin Syah; and Dian M. Setiawan carried out the preparation of the draft manuscript. All authors participated in the review of the results and provided their approval for the final version of the manuscript.

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