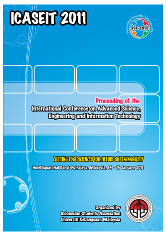
Proceeding of the International Conference on Advanced Science, Engineering and Information Technology 2011

Hotel Equatorial Bangi-Putrajaya, Malaysia, 14 - 15 January 2011

ISBN 978-983-42366-4-9



Ranking Fuzzy Numbers and Its Application to Products Attributes Preferences

Lazim Abdullah and Nor Nashrah Ahmad Fauzee

Department of Mathematics
Faculty of Science and Technology
University of Terengganu Malaysia
21030 Kuala Terengganu, Malaysia
Tel.:+609 6683335, E-mail: lazim_m@umt.edu.my

***Abstract*— Ranking is one of the widely used methods in fuzzy decision making environment. The recent ranking fuzzy numbers proposed by Wang and Li is claimed to be the improved version in ranking. However, the method was never been simplified and tested in real life application. This paper presents a four-step computation of ranking fuzzy numbers and its application in ranking attributes of selected chocolate products. The four steps algorithm was formulated to rank fuzzy numbers and followed by a testimony in sensory evaluation of four selected chocolate cake products that presently available in Malaysian market. Data in form of linguistic terms were collected from thirty eight judges at Terengganu State of Malaysia. Decisions were made based on the centroid point ($\bar{x}, \bar{y}$), where the degree of representative location ($\bar{x}$) is higher than average height ($\bar{y}$). These points permit to characterize the evaluation behaviour of each attribute of the chocolate cake products. The attributes of chocolate cake products then were successfully ranked. according to the selected attributes. The ranking shows the feasibility of the proposed stepwise computation in real application.**

***Keywords*— Centroid points, fuzzy numbers, ranking, product preference**

I. Introduction

One of the most important concepts in fuzzy theories is fuzzy numbers. Fuzzy numbers were studied by [1] as an effort to complement the fuzzy sets theory introduced by [2]. Since its inception, many researchers introspected fuzzy numbers and tried to relate with decision making by proposing methods of ranking fuzzy numbers. Ranking fuzzy numbers seemed as one of the important measurements for decision making problem in a fuzzy environment. Numerous studies have explained ranking fuzzy numbers in decision making. For instance, [3] reviewed some methods to rank fuzzy numbers while [4] proposed fuzzy multiple attribute decision making. In 1993, [5] published a paper in which they describe an index for ordering fuzzy numbers. Dias proposed with ranking alternatives by ordering fuzzy numbers [6]. Reference [7] ranked fuzzy numbers with a satisfaction function and about at the same year [8] utilized artificial neutral networks for automatic ranking fuzzy numbers. Reference [9] presented ranking and defuzzification methods based on area compensation and [10] investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights. Reference [4] have surveyed the existing methods and classified them into four categories. The categories were preference relation method suggested by [11], fuzzy mean and spread method discussed by [12], fuzzy scoring methods pointed out by [13], and linguistic method offered by [14]. In 1998, [15] made a breakthrough paper to suggest method a centroid point-based distance for ranking fuzzy numbers.

Among the existing ranking method, a few methods are claimed to be efficient [16]-18]. These methods generally determine the order of fuzzy numbers by considering the concept of area different. However, [19] argued that some of the above methods were difficult to implement on grounds of computational complexity and others are counterintuitive or not discriminating enough. They also discovered that many methods have different outcomes on the same problem. In their review of ranking fuzzy numbers, they proposed a method of ranking of fuzzy numbers with an area between the centroid and original points. Their method originated from the concepts of [12] and [15]. Reference [19] used the centroid point-based the area between centroid point and original point method in the same order to rank the fuzzy numbers. However, [20] criticised that Chu and Tsao's method suffer from some counter-intuitive results. Due to some problems in ranking proposed by Chu and Tsao, recently [20] proposed a modified area method to rank fuzzy numbers. The revised method of ranking fuzzy numbers claimed successfully overcomes the problems aroused in [19]. Several hypothetical examples were presented to illustrate the proposed method. However, to the best authors' knowledge the method has not been experienced in real life problems. In addition, [20] also claimed that the revised method is simpler than Chu and Tsao's method for ranking of fuzzy numbers. The latter's point has never been put forward any attempt to simplify steps or computation procedures. Based on the significance of fuzzy numbers in decision making and the claimed promising method proposed by [20], the present paper proposes a four-stepwise computation for ranking fuzzy numbers and demonstrates the applicability of the algorithm in preference of chocolate cake products from a sensory evaluation experiment.

This paper proceeds as follows. For the sake of clarity, the related concepts of fuzzy theories are presented in Section II. Stepwise computation of ranking fuzzy numbers with an area between the centroid point and original point is explicated in Section III. A sensory evaluation experiment to rank the selected attributes of chocolate cake products is presented in Section IV. Finally, the paper is concluded in Section V.

II. Preliminaries

In this section the basic notions of fuzzy sets and fuzzy numbers are presented. These notions are expressed as follows.

Definition 2.1 [2]

Let U be a universe set. A fuzzy set A of U is defined by a membership function $f_A(x)\rightarrow[0,1]$, where $f_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 2.2 [21]

A fuzzy subset A of universe set U is normal if and only if $\sup_{x \in U} f_A(x)=1$, where U is the universe set.

Definition 2.3 [21]

A fuzzy subset A of universe set U is convex if and only if $f_A(\lambda x+(1-\lambda)y)\geq(f_A(x) \wedge f_A(y))$, $\forall x,y \in U$, $\forall \lambda \in[0,1]$, where $\wedge$ denotes the minimum operator.

Definition 2.4 [21]

A fuzzy set A is a fuzzy number if and only if A is normal and convex on U.

Definition 2.5 [22]

A triangular fuzzy number A is a fuzzy number with a piecewise linear membership function f_A defined by:

$$f_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3, \\ 0, & otherwise \end{cases}$$

which can be denoted as a triplet (a_1,a_2,a_3).

Definition 2.6 [22]

A trapezoidal fuzzy number A is a fuzzy number with a membership function f_A defined by:

$$f_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & a_1 \le x \le a_2, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a quartet (a_1,a_2,a_3,a_4).

Definition 2.7 [22]

An extended fuzzy number A is described as any fuzzy subset of the universe set U with membership function f_A defined as follows:

(a) f_A is a continuous mapping from U to the closed interval $[0,\omega]$, $0<\omega\leq1$.

(b) $f_A(x)=0$, for all $x \in(-\infty,a_1]$.

(c) f_A is strictly increasing on $[a_1,a_2]$.

(d) $f_A(x)=\omega$, for all $x \in[a_2,a_3]$, as ω is a constant and $0<\omega\leq1$.

(e) f_A is strictly decreasing on $[a_3,a_4]$.

(f) $f_A(x)=0$, for all $x \in[a_4,\infty)$.

In these situations, a_1, a_2, a_3 and a_4 are real numbers.

Definition 2.8 [22]

The membership function f_A of the extended fuzzy number A is expressed by

$$f_A = \begin{cases} f_A^L(x), & a_1 \le x \le a_2, \\ \omega, & a_2 \le x \le a_3, \\ f_A^R(x), & a_3 \le x \le a_4, \\ 0, & \text{otherwise}, \end{cases}$$

where $f_A^L : [a_1,a_2] \rightarrow [0,\omega]$ and $f_A^R : [a_3, a_4] \rightarrow [0,\omega]$.

Based on the basic theories of fuzzy numbers, A is a normal fuzzy number if $\omega=1$, whereas A is a non-normal fuzzy number if $0<\omega<1$.

III. COMPUTATIONS

The steps are proposed based on [20] and [19] in ranking fuzzy numbers.

Step 1. Define triangular fuzzy numbers and its respective linguistic variables

The triangular fuzzy numbers is based on a three-value judgement of a linguistic variable. The minimum possible value is denoted as a_1, the most possible value denoted as a_2 and the maximum possible value denoted as a_3.

Step 2. Delineate Inverse Function

The inverse function of f_A^L exists as $f_A^L : [a_1,a_2] \rightarrow [0, \omega]$ is continuous and strictly increasing, and the inverse function of f_A^R exists as $f_A^R : [a_3, a_4] \rightarrow [0,\omega]$ is continuous and strictly decreasing. The inverse functions I_A^L and I_A^R of f_A^L and f_A^R respectively. Since f_A^L : $[a_1,a_2] \rightarrow [0,\omega]$ is continuous and strictly increasing, $I_A^L : [0, \omega] \rightarrow [0,1]$ is also continuous and strictly increasing. Similarly, $f_A^R : [a_3, a_4] \rightarrow [0,\omega]$ is continuous and strictly decreasing, and thus $f_A^R : [0,\omega] \rightarrow [a_3,a_4]$ is continuous and strictly decreasing as well. In short, I_A^L and I_A^R are continuous on $[0,\omega]$, so I_A^L and I_A^R exist.

Step 3. Establish Centroid Point $(\bar{x}, \bar{y})$.

The centroid point of a fuzzy number A corresponded to a value $\bar{x}$ on the horizontal axis and a value $\bar{y}$ on the vertical axis. The centroid point $(\bar{x}(A), \bar{y}(A))$ of a fuzzy number A was defined as

$$\bar{x}(A) = \frac{\int_{a1}^{a2} x f_A^L(x)dx + \int_{a2}^{a3} x dx + \int_{a3}^{a4} x f_A^R(x)dx}{\int_{a1}^{a2} f_A^L(x) + \int_{a2}^{a3} dx + \int_{a3}^{a4} f_A^R dx}$$

and

$$\bar{y}(B) = \frac{\int_0^{\omega} y I_{\tilde{B}}^L(y)dy + \int_0^{\omega} y I_{\tilde{B}}^R(y)dy}{\int_0^{\omega} I_{\tilde{B}}^L(y)dy + \int_0^{\omega} I_{\tilde{B}}^R(y)dy}$$

where f_A^L and f_A^R were left and right membership functions of A respectively, and f_A^L and f_A^R were inverse functions of I_A^L and I_A^R respectively [23] [5].

Step 4. Decision Rule

For two fuzzy numbers A and B, they had the following relation:

If $\bar{x}(A) > \bar{x}(B)$, then $A > B$.
If $\bar{x}(A) < \bar{x}(B)$, then $A < B$
If $\bar{x}(A) = \bar{x}(B)$, then
if $\bar{y}(A) > \bar{y}(B)$, then $A > B$;
else if $\bar{y}(A) < \bar{y}(B)$, then $A < B$;
else $\bar{y}(A) = \bar{y}(B)$, then $A = B$.

In short, A and B are ranked based on their $\bar{x}$'s values if they are different. In the case they are equal, they are ranked by comparing their $\bar{y}$'s values.

With the above stepwise computation of ranking method, an experiment on sensory evaluation was carried out and described in the following section.

IV. APPLICATION

Thirty eight chocolate cake consumers from Terengganu State, Malaysia were selected as judges on the selected attributes. The quality attributes selected for the sensory evaluation were colour (C), texture (T), humidity (H), flavour (V) and sweetness (S) of the selected chocolate product. A twenty-item questionnaire was used for the evaluation. Judges were told the purpose of the research and quality attributes of chocolate cakes. Judges were asked to evaluate the score for the first attribute, colour (C) followed by four other attributes. All of them need to tick (✓) mark in the respective linguistic value in scale 0 to 1 for each of the quality attributes of the sample after evaluating the samples based on their own criteria and likings regarding chocolate cakes. Four brands of chocolate cakes were judged and named as C cake (1), HME cake (2), SB cake (3), and Sid cake (4). For the purpose of report, the real name of four branded cakes was not disclosed.

Analyses of sensory data of the products were executed based on the algorithm as prescribed in Section 3. The attribute of colour (C) is chosen as an example of these analyses.

Step 1. Define Triangular Fuzzy Numbers and linguistics variables.

Five scales of linguistic variables from excellent to not satisfactory were defined. Definition for each linguistic variable and their fuzzy value were based on work done by Jaya and Das [24]. Linguistic variables data from the questionnaire were expressed in fuzzy value as shown in Table 1.

Table I
Complacency level in linguistic value and fuzzy value.

Linguistic value	Fuzzy number
Excellent	0.00
Good	0.25
Medium	0.50
Fair	0.75
Not satisfactory	1.00

The complacency levels in fuzzy values are utilized to translate the responses from questionnaire into fuzzy values. The arithmetic mean of fuzzy value for 38 judges is obtained as 0.30. Therefore triangular fuzzy number (TFN) for C_1 is defined as (0.10, 0.30, 0.50). Triangular fuzzy numbers for the other three products C_2, C_3, and C_4 over the attribute of colour are defined as Table II.

Table II Triangular fuzzy numbers(TFN) of colour attribute

Attribute	TFN
C_1	(0.10, 0.30, 0.50).
C_2	(0.08, 0.28, 0.48)
C_3	(0.23, 0.43, 0.63)
C_4	(0.42, 0.62, 0.82)

The triangular fuzzy numbers can be expressed in membership functions that eventually needed to find the inverse functions.

Step 2. Delineate Inverse Function

The inverse functions for f^L_{Ci}, f^R_{Ci} for their respective products are shown in the Table III.

Table III
The inverse function of f^L_{Ci} and f^R_{Ci}.

Colour attribute	I^L_{Ci}	I^R_{Ci}
C_1	$0.10+0.2y$	$0.5-0.2y$
C_2	$0.08+0.2y$	$0.48-0.2y$
C_3	$0.23+0.2y$	$0.63-0.2y$
C_4	$0.42+0.2y$	$0.82-0.2y$

Step 3. Establish Centroid Point ($\bar{x}$, $\bar{y}$)

The value of centroid point ($\tilde{x}, \tilde{y}$) can be obtained by using Maple 9.5 package, and the results for Colour (C_1) attributes are presented in Table IV. Due to the limited space, the details of calculations are not shown in this paper.

Table IV
Centroid points of colour attribute for all the sample products.

Colour attribute	$\tilde{x}$	$\tilde{y}$
C_1	0.3957446809	0.5000000000
C_2	0.2800000000	0.5000000000
C_3	0.4300000000	0.5000000000
C_4	0.6200000000	0.5000000000

Step 4. Decision Rule

Based on the proposed decision, ranking of fuzzy numbers for all the chocolate cake products eventually give the solution to find out the best market chocolate cake products. The ranking order for the attribute of colour is $C_2 \prec C_1 \prec C_3 \prec C_4$. The notion '$\prec$' is refereed as 'preferred to'. For the attribute of colour, the product HME cake (2) is the most preferred product among the four.

With the similar fashion, the analyses for the other four attributes of products are executed. Finally, the ranking of other attributes are $T_3 \prec T_1 \prec T_4 \prec T_2$, $H_1 \prec H_3 \prec H_2 \prec H_4$, $V_3 \prec V_1 \prec V_2 \prec V_4$ and $S_1 \prec S_3 \prec S_4 \prec S_2$. The most preferred products according to the attributes can be drawn from the rankings. This analysis is not meant to seek the best product bounded by all attributes but rather to test the proposed four-stepwise computations. The uncertainty in sensory evaluation allows the flexibility in fuzzy numbers to come up with ranking. Fuzzy numbers successfully used for representing the numerical quantities in a vague environment.

V. CONCLUDING REMARKS

This paper has proposed the step-wise computation to ease complexity in ranking fuzzy numbers. The four-step algorithm was developed after considering the potential applications of ranking fuzzy numbers in real case of decision making. The basis for this paper is the theoretical work of Wang and Li, who exploring the ranking fuzzy numbers in decision making. An application to test the algorithm in sensory evaluation has been experimented. The centroid point ($\tilde{x}, \tilde{y}$) of attributes determined the acceptance level of products. This algorithm was successfully used in ranking of chocolate cake products based on its respective attributes. There was no specific product shows superior than the other but at least the sensory evaluation unveiled the importance of single attribute in consumers' preferences via robustness of the algorithm. Further analysis are still unlock in finding the best product with multiple attributes. Ranking of the product based on single attribute becomes a testimony to the algorithm at least with the present experiment data. This study offers a new extension in application of ranking fuzzy numbers. Further experimental investigations are needed to amplify the application of ranking fuzzy numbers and hopefully broaden the horizon of computing.

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