

Integrated Models of Non-Permanent Information Warfare

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Abstract—In the paper, a new Integrated Mathematical Model of Information warfare is built. In the suggested model, by selecting continuous intensity coefficients of aggressiveness of the conflicting parties and the peacemaking activity of a third party, it is possible to describe the process of Non-Permanent Information Warfare with restrictions. The Non-Permanence of Information Warfare is due to an increase in the information confrontation between the two sides over a certain period. In the modeling, Non-Permanent Information Warfare has highlighted a particular boundary value problem. The existence of the solution of the special boundary value problem determines the controllability of the Non-Permanent Information Warfare by the peacekeeping side. The task of the peacekeeping side is to end Information Warfare by the conflicting sides, i.e., to stop them from spreading negative information against each other. By using a computer experiment, various modes of Information Warfare have been studied, depending on the strategies of the sides. In particular, the regime of mutual attenuation of the parties is considered, when the conflicting parties simultaneously increase the amount of information distributed by a certain period and then reduce them. The regime of mutual aggravation is also considered. For each mode of development of Non-Permanent Information Warfare, the problem of controllability is separately studied, and a forecast of development of the process of Information Warfare for different values of parameters of the system is given. For the peacekeeping side management, parameters are proposed - coefficient peacekeeping activity and the level of Information Technology.

Keywords—non-permanent information warfare; escalation; attenuation; information attack; integrated mathematical model; computer model; computer experiment; controllability.

I. INTRODUCTION

The exacerbation of the Information Confrontation, which may be caused by some event in time, causes a specific scientific interest in Modeling Information Warfare [1], [2]. In the article, we consider the system of Information Warfare in which three participants - three sides - are identified. Two of them are antagonistic against each other and disseminate the flows of information characteristic of the confrontation. The third side is a peacekeeping side, which also distributes the flow of information, but aimed at reconciling the antagonistic parties [3]-[5]. Some models of information flow of Information Warfare consider the restriction on information flows, due to the level of development of the parties' Information Technologies [6],[7].

In the computer and mathematical models of the Information Flow Streams of the Information Warfare, we

also consider the number of adherents - those people who perceived the flows of antagonistic information and themselves began to disseminate this information at an interpersonal level. Thus, we deal with an integrated model, which describes both the flows of information and the adherents, who perceived this information [8]. It should be noted that in the modeling of the Information Warfare, in which adepts are in focus, many scientists work, and they obtain interesting results [2],[9],[10]. Moreover, the model of linguistic-information confrontation is considered [11].

Note that, on the observed interval of time $[0;T]$ for integrated models of Information Warfare [8] there are no features. This means that any moment $t \in [0;T]$ is no different from any other moment in time $\xi \in [0;T]$. Meanwhile, often the informational confrontation is confined to some event tied to a certain point of time [1]. For

example, opposing parties may wage Information Warfare to succeed in the presidential election, which is scheduled at a time $\tau \in [0; T]$. As a rule, in these cases, the antagonistic parties increase the intensity of Information Attacks by the time point τ , and then after some time, they either reduce the intensity of attacks or increase it. A decrease in the intensity of Information Attacks occurs if the parties recognize the election results; if the parties or at least one of the parties does not recognize the election results, the intensity of Information Attacks increases. The parties or one of the parties can prepare an informational background for the revolutionary development of events. Thus, the Information Warfare does not proceed permanently but is confined to a certain event, and after the completion of this event, the Information Warfare may fade away or gain a new impulse and continue in the Escalation mode. In the latter case, the role and responsibility of the peacekeeping side are especially growing. The question is whether the peacekeeping side will be able to prevent and at what activity the Information Warfare taking place in an Escalation regime.

In this work, we propose an integrated model of the Non-Permanent Information Warfare and find out the question of its controllability. Unlike conventional models in the integrated model of Non-Permanent Information Warfare, we present more active involvement in the peacekeeping side. The presented work was supported by grant YS17_78 of the Shota Rustaveli National Science Foundation of Georgia.

II. MATERIALS AND METHODS

At the first stage, we build an integrated mathematical model of Non-Permanent Information Warfare. To do this, we used one of the models proposed in [8] and modify it. As in [1] the process of Non-Permanent Information Warfare was observed on the time interval $[0; T] \subset R$, where R is the set of real numbers. On this segment, we define the time moment $\tau \in [0; T]$, where $0 < \tau < T$, and in which some event takes place, to this event, antagonistic parties are trying to gain superiority in this Information Warfare. It is natural to assume that by the time point, τ the antagonistic sides increase the intensity of information attacks. We denote by $N_1(t)$ the amount of information, that the first antagonistic side is disseminating at the t point in time. Let the second antagonistic side at a time point t spread $N_2(t)$ amount of information. The third, peacekeeping party participating in the Information Warfare, is disseminating information calling on the antagonistic parties to stop information attacks in an equal volume to $N_3(t)$. In constructing the mathematical model, we assume that the antagonistic sides adhere only to their tactics of conducting Information Attacks, not considering the activity of the other antagonistic side. At the same time, each of the antagonistic parties, to some extent, listens to the calls of the third peacemaking side. The willingness to listen to the third side calls the peacekeeping readiness of the party. We measured and denoted it by $\beta_1(t)$ and $\beta_2(t)$ respectively, for the first and second sides. The intensity of the Information

Attacks of the antagonistic parties depends on the aggressiveness index of each of the parties is $\alpha_1(t)$ and $\alpha_2(t)$ respectively. The activity of the third side also depends on its peacekeeping intensity provoked by each of the antagonistic parties and which has significance $\gamma_1(t)$ and $\gamma_2(t)$. The development level of the information technology of the parties we determined by their ability to disseminate the maximum amount of information, and we denoted it by I_1 and I_2 , respectively for the first and second side. The level of development of third-side Information Technologies is denoted by I_3 . The process of disseminating information among adepts - supporters of the information received was first used in the Samarskiy-Mikhailov model [9]. We denote by $x(t)$ the number of people (adepts - adherents) who perceived the information flows $N_1(t)$ and then began to distribute them at the interpersonal level in the community in which the x_p is a member. Similarly, let $y(t)$ denotes the number of people (adepts) who perceived the information flows $N_2(t)$ and then began to distribute them at the interpersonal level in the community in which y_p are members. For the first side $\delta_1 N_1(t)$ means the intensity of the dissemination $N_1(t)$ of information by the general method, for example through the media, and $\mu_1 N_1(t)$ - the intensity of the dissemination $N_1(t)$ of information at the interpersonal level. Similarly for the second side $\delta_2 N_2(t)$ means the intensity of the dissemination of the information $N_2(t)$ by the general method, for example through the media, and $\mu_2 N_2(t)$ - the intensity of dissemination of information $N_2(t)$ at the interpersonal level.

Now the Integrated Mathematical Model of the Information Warfare in the form of Cauchy problem for a system of ordinary differential equations, we can write as follows:

$$\begin{cases}
\frac{d}{dt} N_1(t) = \alpha_1(t)(N_1(t) + v_1 x(t)) \times \\
\times \left(1 - \frac{N_1(t)}{I_1}\right) - \beta_1(t) N_3(t), \\
\frac{d}{dt} N_2(t) = \alpha_2(t)(N_2(t) + v_2 y(t)) \times \\
\times \left(1 - \frac{N_2(t)}{I_2}\right) - \beta_2(t) N_3(t), \\
\frac{d}{dt} N_3(t) = (\gamma_1(t) N_1(t) + \gamma_2(t) N_2(t)) \times \\
\times \left(1 - \frac{N_3(t)}{I_3}\right), \\
\frac{dx(t)}{dt} = N_1(t)(\delta_1 + \mu_1 x(t))(x_p - x(t)), \\
\frac{dy(t)}{dt} = N_2(t)(\delta_2 + \mu_2 y(t))(y_p - y(t)). \\
\begin{cases} N_1(0) = n_{10}, N_2(0) = n_{20}, N_3(0) = n_{30}, \\ x(0) = x_0, y(0) = y_0. \end{cases}
\end{cases} \quad (1)$$

The initial conditions of the Cauchy problem are given in system (2), where the first line contains the number of information flows of each of the parties involved in the Information Warfare at the initial time $t=0$; and the second line shows the number of adherents of the antagonistic parties at the initial moment of time - for the first side it is equal x_0 , and for the second side it is equal to y_0 .

Meanwhile, if the task of information security of the system is solved - i.e., completion of the Information Warfare, then under certain conditions activity of the peacekeeping side can achieve it. Specifically, with the proper activity of the peacekeeping side, the antagonistic sides can stop information attacks, and thereby, the Information Warfare would end. The completion of an information attack by the party was considered the case when it stops distributing aggressive information flows, i.e., at some point in time $t^* \in [0; T]$ for the first side $N_1(t^*)=0$ and for the second side $N_2(t^{**})=0$ at some time point $t^{**} \in [0; T]$. Moreover, each of the antagonistic parties can complete information attacks at different points of time which are not predetermined. If these remarks are given in the mathematical model of the end of the Information Warfare, the following conditions must be introduced: are there any arbitrary points in time $t^*, t^{**} \in [0; T]$ at which:

$$\begin{cases} N_1(t^*)=0, N_1(t) \leq 0 \quad \forall t \in [t^*; T], \\ N_2(t^{**})=0, N_2(t) \leq 0 \quad \forall t \in [t^{**}; T]. \end{cases} \quad (3)$$

Thus, the mathematical model of ending the Information Warfare from the Cauchy problem turns into a problem with special boundary conditions (1), (2), (3), which sometimes is called Chalker problem [12]-[14]. We can obtain an Integrated Mathematical Model of Non-Permanent Information Warfare from the model (1) - (3) by selecting special functions for the aggressiveness index of the antagonistic sides. We assume that the activity coefficients of adherents correspond to the aggressiveness of the respective parties. We call model (1), (2) ignoring the enemy, when the parties carry out information attacks inside a closed system, like an echo-camera, and such systems are objects of study [10].

III. RESULTS AND DISCUSSION

Let us look at a possible scenario for the development of Non-Permanent Information Warfare. As it was noted, in the Non-Permanent Information Warfare at some point in time, some event takes place, to which the increase in the intensity of Information Attacks is timed. However, after this event, the Information attacks decrease or increase. Separately consider for the generalized model these two scenarios of the development of the course of the Information Warfare.

A. Integrated Model of Non-Permanent Information Warfare with Mutual Attenuation

To obtain a Mathematical Model Non-Permanent Information Warfare with mutual attenuation from (1) - (3) model, we select in the system (1) the function of the aggressiveness index of antagonistic sides. Since the information attacks of the antagonistic parties increase before the moment of time τ and then decrease, for example, to zero, it is logical to select the functions of the aggressiveness indices in the system (1) in the form of bell-shaped functions with a peak in the vicinity of the time point τ or at this point itself. As such functions, for example, some trigonometric functions and parabolas can be selected at a specified interval.

Let us look at the model task for the system (1), consider specific functions, and assume that $\alpha_1(t) = \alpha_2(t) = \alpha(t)$. Let us suppose that there is an inclusion $[0; 2\tau] \subseteq [0; T]$.

Then we can suppose, that $\alpha(t) = A \left(\sin \left(\frac{\pi}{2\tau} t \right) + \varepsilon \right)$, where

A, ε are positive constants. In this case, we assume that ε is an arbitrarily small number. Suppose we have:

$$A_1 = const, \quad \beta_1(t) = \beta_2(t) = \beta = const,$$

$$\gamma_1(t) = \gamma_2(t) = \gamma(t) = A_1 \left(\sin \left(\frac{\pi}{2\tau} t \right) + \varepsilon \right), \quad v_1 = const,$$

$$v_2 = const, \quad \delta_1 = \delta_2 = \delta(t) = A_2 \left(\sin \left(\frac{\pi}{2\tau} t \right) + \varepsilon \right),$$

$$\mu_1 = \mu_2 = \mu(t) = A_3 \left(\sin \left(\frac{\pi}{2\tau} t \right) + \varepsilon \right).$$

Then system (1) has the form:

$$\begin{cases}
\frac{d}{dt} N_1(t) = \alpha(t)(N_1(t) + v_1 x(t)) \times \\
\times \left(1 - \frac{N_1(t)}{I_1}\right) - \beta N_3(t), \\
\frac{d}{dt} N_2(t) = \alpha(t)(N_2(t) + v_2 y(t)) \times \\
\times \left(1 - \frac{N_2(t)}{I_2}\right) - \beta N_3(t), \\
\frac{d}{dt} N_3(t) = \gamma(t)(N_1(t) + N_2(t)) \times \\
\times \left(1 - \frac{N_3(t)}{I_3}\right), \\
\frac{dx(t)}{dt} = N_1(t)(\delta(t) + \mu(t)x(t)) \times \\
\times (x_p - x(t)), \\
\frac{dy(t)}{dt} = N_2(t)(\delta(t) + \mu(t)y(t)) \times \\
\times (y_p - y(t)).
\end{cases} \quad (4)$$

The Cauchy problem (4), (2) was investigated using a computer experiment. We also are interested in the controllability of the system, i.e., the ability to transfer the system from state (2) to state (3) using selection $\gamma(t)$. The Information Warfare system is described using an integrated mathematical model (4). We carried out the computational experiment in the environment of the MatLab Application Package. For a Non-Permanent Information Warfare with mutual attenuation, as shown by a computer experiment in the general form, the problems of system controllability are not worth it. Since the Information Attacks of the first and second sides become equal to zero, i.e., the Information Warfare ends, with a low peace activity of a third side, see Fig. 1.

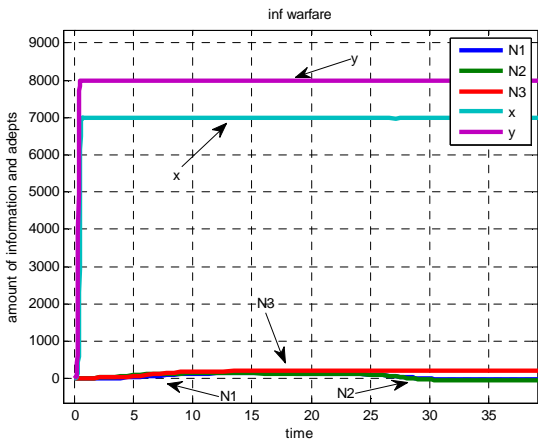


Fig. 1 The activity of the parties in the Integrated Model of the Non-Permanent Information Warfare with mutual attenuation

So, for example, with the observation interval $[0; 40]$, the initial conditions - $N_1(0) = 10$, $N_2(0) = 15$, $N_3(0) = 19$, $x(0) = 0.05$, $y(0) = 0.03$ и $A = 0.9$, $A_1 = 0.5$, $A_2 = 0.07$, $A_3 = 0.006$, $v_1 = 0.001$, $v_2 = 0.005$, $\varepsilon = 0.001$, $\tau = 12$,

$\beta = 0.05$, $\gamma = 0.05$, $I_1 = 155$, $I_2 = 150$, $I_3 = 200$, $x_p = 7000$, $y_p = 8000$ the first antagonistic side terminates the Information Attack at a time $t^* = 29.8748$, and the second at a time $t^{**} = 28.7357$, see Fig. 2.

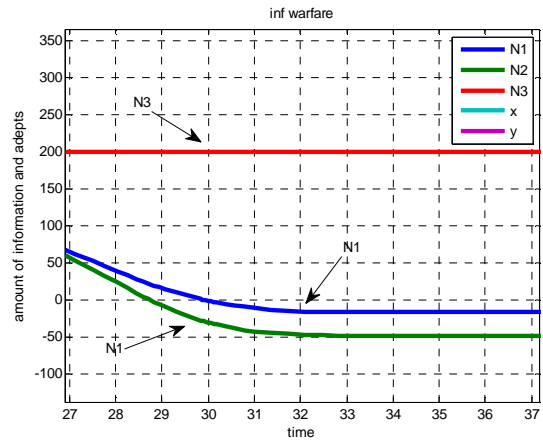


Fig. 2 Completion of information attacks by antagonistic parties

It should be noted that the number of adherents of antagonistic parties pretty soon reaches the population of the parties, see Fig. 3.

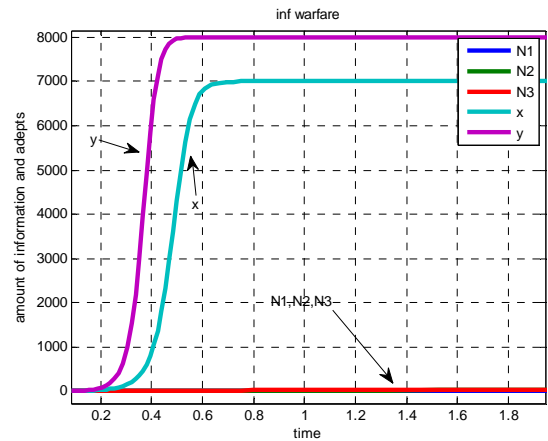


Fig. 3 The dynamics of growth in the number of adherents of antagonistic parties

For a computer experiment, we used program code with the ode15s solver, calculated for the numerical solution of hard systems. The code is shown in Listing 1.

Listing 1. Program code to solve the model Non-Permanent Information Warfare with mutual attenuation and peacekeeping Activity

```

gmc_m_np_iw.m
n0=[10 15 19 0.05 0.03];
[T,Y]=ode15s('zat_np_gmc_miw',[0
40],n0);plot(T,Y,'LineWidth',3)
title('inf warfare')
xlabel('time')
ylabel('amount of information and
adepts')
legend('N1','N2','N3','x','y')
grid on

```

zat_np_gmc_miw.m

```

%ode- right side of the system i
function dndt=zat_np_gmcmiwit(t,n)
dndt=zeros(5,1); ep=0.001;v1=0.001;
v2=0.005;ut=12; i1=155; i2=150;i3=200;
a=0.9; a1=0.5; a2=0.07; a3=0.006;
xp=7000; yp=8000;b=.05;
dndt(1)=a*(sin(pi*t/(2*ut))+ep)*(n(1)+v1
*n(4))*(1-n(1)/i1)-b*n(3);
dndt(2)=a*(sin(pi*t/(2*ut))+ep)*(n(2)+v2
*n(5))*(1-n(2)/i2)-b*n(3);
dndt(3)=a1*(sin(pi*t/(2*ut))+ep)*(n(1)+n
(2))*(1-n(3)/i3);
dndt(4)=n(1)*(sin(pi*t/(2*ut))+ep)*(a2+a
3*n(4))*(xp-n(4));
dndt(5)=n(2)*(sin(pi*t/(2*ut))+ep)*(a2+a
3*n(5))*(yp-n(5));
end

```

A Non-Permanent Information Warfare with mutual attenuation can actually end even when the peacekeeping side does not show any activity, while the ending time of the Information Warfare increases. So, for example, if we exclude the activity of a third side in the program code given in Listing 1 and set its initial value to zero, i.e. we assume that $\gamma(t)=0.0$ and $N_3(0)=0.0$, then according to the results of solving the model (4), (2) it is clear that the first antagonistic side terminates the Information Attack at the time $t^* = 40.4874$, and the second at the time $t^{**} = 38.3545$, see Fig. 4.

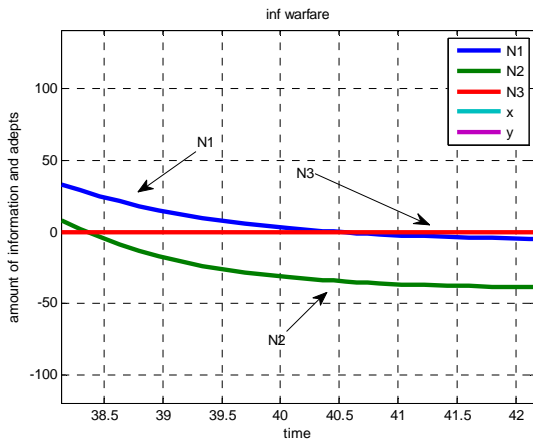


Fig. 4 Non-Permanent Information Warfare with mutual attenuation, without peacekeeping activity

The numerical experiment concluded that the Non-Permanent Information Warfare, with mutual attenuation described by the model (4), (2) ends even without any effort on the part of the peacekeeping force. And the effort of the peacekeeping side is necessary only if the task must end the Information Warfare strictly by a certain time point.

B. Integrated Model of Non-Permanent Information Warfare with Escalation

An Integrated Model of Non-Permanent Information Warfare with Escalation can be obtained from (1) - (3) models, if we select in the system of ordinary differential equations (1) the aggressiveness functions of the antagonistic sides. Since the information attacks of the antagonistic parties increase up to the time point τ and this process continues further, it is logical that the intensity index

in the system (1) should be monotonically increasing (non-decreasing) functions, as was proposed in [1]. As such, functions, for example, can choose an exponential function or a power function with an odd exponent.

We also investigated the problem of the Cauchy of Non-Permanent Information Warfare with Escalation using a computer experiment. In this case, we are interested in the controllability of the system, i.e., the ability to transfer the system from state (2) to state (3) by selecting values for $\gamma(t)$, I_3 . We carried out the computational experiment again in the MatLab environment. We investigated the integrated model of Non-Permanent Information Warfare with Escalation, considering two scenarios. In the first scenario, both antagonistic sides resort to Escalation. In the second scenario, only one of the party's resorts to Escalation, while the second is either neutral or acts in the damping mode. We separately consider each scenario of the development of the event.

1) *An integrated model of non-permanent information warfare with mutual escalation:* In system (4), we consider specific functions and assume that $\alpha(t) = A((t-\tau)^3 + \tau^3)$, where A is a positive constant. For peacekeeping activity, we assume that it is also growing monotonously: $\gamma(t) = A_1((t-\tau)^3 + \tau^3)$, where A_1 is a positive constant. $\delta(t) = A_2((t-\tau)^3 + \tau^3)$, $\mu(t) = A_3((t-\tau)^3 + \tau^3)$, where A_2 , A_3 are also positive constants.

Now, the Cauchy problem (4), (2) were solved numerically. For the observation interval $[0; 40]$, and the initial conditions $N_1(0) = 10$, $N_2(0) = 15$, $N_3(0) = 19$, $x(0) = 0.05$, $y(0) = 0.03$, values of constants $A = 0.9$, $A_1 = 0.5$, $A_2 = 0.07$, $A_3 = 0.006$, $v_1 = 0.001$, $v_2 = 0.005$, $\varepsilon = 0.001$, $\tau = 12$, $\beta = 0.05$, $\gamma = 0.05$, $I_1 = 155$, $I_2 = 150$, $I_3 = 200$, $x_p = 7000$, $y_p = 8000$. The Information Warfare is not ending, the third side is unable to repay it. See Fig. 5.

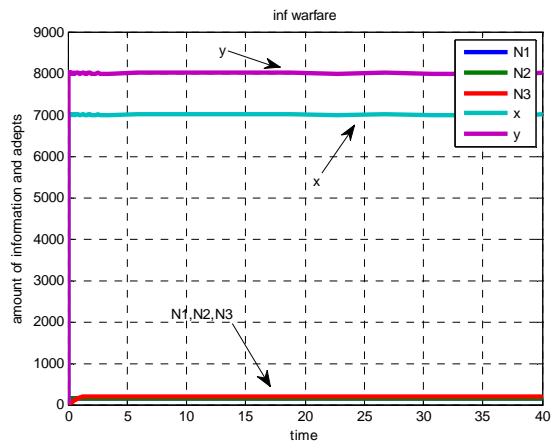


Fig. 5 Non-Stop Non-Permanent Information Warfare with Mutual Sharp Escalation

With these values of the model, the first and second antagonistic sides very soon reach their ultimate mode of Information Attacks. A third side also goes to the limit of the

dissemination of information, due to the level of Information Technology. Adepts go to their maximum, too. See Fig. 6.

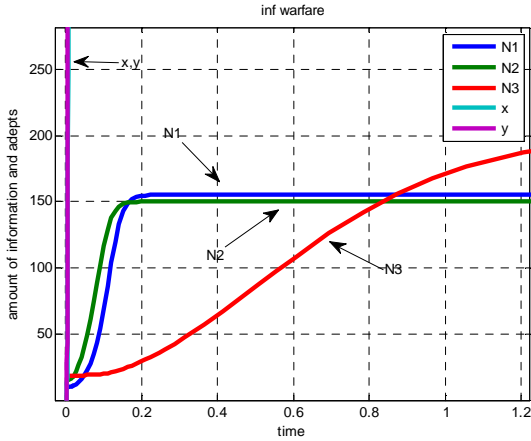


Fig. 6 Withdrawal of the parties and adherents in the Non-Stop Non-Permanent Information Warfare with Mutual Sharp Escalation

The problem of controllability during sharp mutual exacerbations is quite complex and difficult to achieve. For example, with increases up to 60,000 units, and the level of technological levels up to 5700 units, information attacks can be stopped. Only the first side, while the second side does not succumb to the influence of the peacekeeping side and continues to information attacks. Moreover, only with meanings $A_1 = 75000$ and $I_3 = 15000$ the Information Warfare would be ended: the first and second antagonistic parties, respectively, stop the Information Attacks at time points: $t^* = 0.0199$ and $t^{**} = 0.0761$, see Fig. 7.

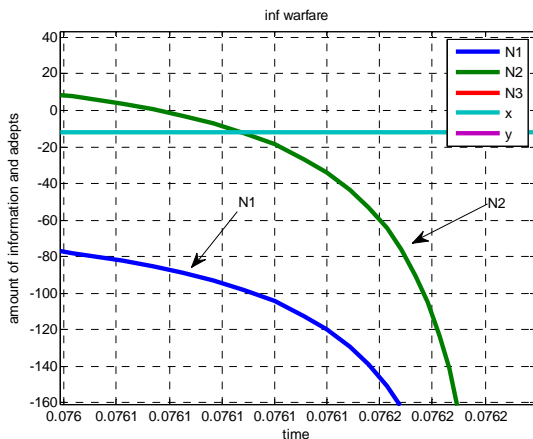


Fig. 7 By the efforts of the high activity of the peacekeeping side, it is possible to end the Non-Permanent Information Warfare with a mutual sharp escalation

The program code for the computer model Non-Permanent Information Warfare with mutual sharp escalation is given in Listing 2. Listing 2. Program Code for Solving the Integrated Model of Non-Permanent Information Warfare with Mutual Sharp Escalation.

```
gmcm_np_iw_oo.m
n0= [10 15 19 0.05 0.03];
[T, Y] =ode15s ('oo_np_gmcmiw', [0 40],
n0); plot (T, Y, 'LineWidth',3)
title ('inf warfare')
xlabel('time')
```

```
ylabel('amount of information and adepts')
legend('N1','N2','N3','x','y')
grid on
oo_np_gmcmiw.m
%ode- right side of the system i
function dndt=oo_np_gmcmiwit(t,n)
dndt=zeros (5,1); ep=0.001; v1=0.001;
v2=0.005;ut=12; i1=155; i2=150;i3=200;
a=0.9; a1=0.006; a2=0.07; a3=0.006;
xp=7000; yp=8000; b=.05;
dndt(1) =a*((t-ut)^3+ut^3)*(n(1)+v1*n(4))*(1-n(1)/i1)-
b*n(3);
dndt(2) =a*((t-ut)^3+ut^3)*(n(2)+v2*n(5))*(1-n(2)/i2)-
b*n(3);
dndt(3) =a1*((t-ut)^3+ut^3)*(n(1)+n(2))*(1-n(3)/i3);
dndt(4) =n(1)*((t-ut)^3+ut^3)*(a2+a3*n(4))*(xp-n(4));
dndt(5) =n(2)*((t-ut)^3+ut^3)*(a2+a3*n(5))*(yp-n(5));
end
```

As a computer experiment shows, the finish of the Information Warfare becomes impossible if the antagonistic parties have sharp Information Attacks close, respectively, to the levels of Information Technology of the parties.

2) *Integrated Model of Non-Permanent Information Warfare with Unilateral Escalation:* The particular interest is the integrated model of Non-Permanent Information Warfare is when one of the parties, after reaching a point in time τ , gradually reduces the volume of Information Attacks, i.e., attenuation of its activity occurs. The other side, after reaching a time point τ , builds up Information Attacks on the contrary, i.e. exacerbates the Information Warfare.

Separately, we consider the case when the first side exacerbates Information Attacks, and the second chooses the attenuation mode and vice versa. At the same time, we are interested in precisely the possibility of selecting various indices of aggressiveness of the antagonistic parties for these cases. Justify this approach, since in paragraph III.B.1) it was established that in the event of a sharp mutual Escalation, the peacekeeping side must make incredible efforts to achieve the end of the Information Warfare. Specifically, as an index of aggressiveness, you can choose,

for example, functions for which $\alpha(t) = A((t-\tau)^3 + \tau^3)$ is an infinitely large higher order and functions slowly changing at infinity. Such functions can be $\ln(t)$, $\ln(\ln(t))$, $\arctan(t)$ and others that are similar. Let us assume that in system (1) $\alpha_1(t) = A_1 \ln(1 + \ln(1+t+\varepsilon))$, $\alpha_2(t) = A_2 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right)$, $\beta_1(t) = \beta_2(t) = \beta = const$, $\gamma_1(t) = \gamma_2(t) = \gamma(t) = A_3 \ln(1 + \ln(1+t+\varepsilon))$, $\delta_1(t) = A_4 \ln(1 + \ln(1+t+\varepsilon))$,

$$\mu_1(t) = A_5 \ln(1 + \ln(1 + t + \varepsilon)), \quad \delta_2(t) = A_6 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right),$$

$$\mu_2(t) = A_7 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right),$$

where $A, A_i (i=1-7), \varepsilon$ are positive constants. Then system (1) can be written in the following way:

$$\begin{cases} \frac{d}{dt} N_1(t) = A_1 \ln(1 + \ln(1 + t + \varepsilon))(N_1(t) + v_1 x(t)) \left(1 - \frac{N_1(t)}{I_1}\right) - \beta_1(t) N_3(t), \\ \frac{d}{dt} N_2(t) = A_2 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right) (N_2(t) + v_2 y(t)) \left(1 - \frac{N_2(t)}{I_2}\right) - \beta_2(t) N_3(t), \\ \frac{d}{dt} N_3(t) = A_3 \ln(1 + \ln(1 + t + \varepsilon)) \times (N_1(t) + N_2(t)) \left(1 - \frac{N_3(t)}{I_3}\right), \\ \frac{dx(t)}{dt} = N_1(t) \ln(1 + \ln(1 + t + \varepsilon)) (A_4 + A_5 x(t)) (x_p - x(t)), \\ \frac{dy(t)}{dt} = N_2(t) \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right) (A_6 + A_7 y(t)) (y_p - y(t)). \end{cases} \quad (5)$$

The Cauchy problem (5), (2) was investigated using a computational experiment. A computer experiment showed that the Non-Permanent Information Warfare given by the integrated model (5), (2) is controllable, i.e., there are such values for $\gamma(t), I_3$, at which the Information Warfare system from the state (2) goes into the state (3).

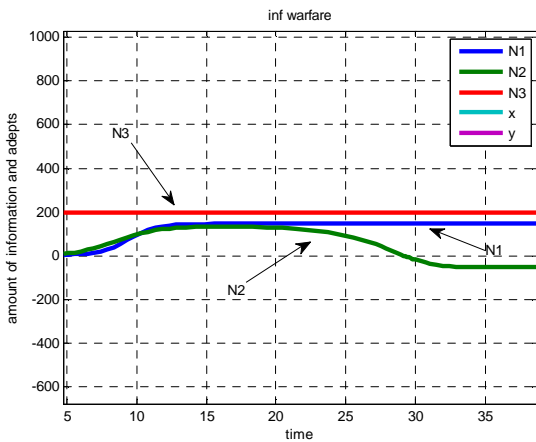


Fig. 8 Non-Permanent Information Warfare with Escalation of the first and the attenuation of the second side, the Information Warfare does not end

For example, if with values: $N_1(0)=10, N_2(0)=15, N_3(0)=19, x(0)=0.05, y(0)=0.03, A_1=0.9, A_2=0.6, A_4=0.5, A_5=0.045, A_6=0.8, A_7=0.075, \tau=5, \beta=0.05, \gamma=0.07 \times \ln(1 + \ln(1 + t + \varepsilon)), v_1=0.001, v_2=0.005, \varepsilon=0.001, \tau=12, I_1=155, I_2=150, I_3=250, x_p=7000, y_p=8000$, the first side does not stop the Information Attack, unlike the second side, see Fig. 7, then with increasing peacekeeping activity up to $\gamma(t) = 7.07 \times \ln(1 + \ln(1 + t + \varepsilon))$ and the technological level of the third side up $I_3 = 400$, then both antagonistic sides complete the Information Attacks, the first side at the time $t^* = 2.5652$ and the second at the time $t^{**} = 2.7997$, see Fig. 9.

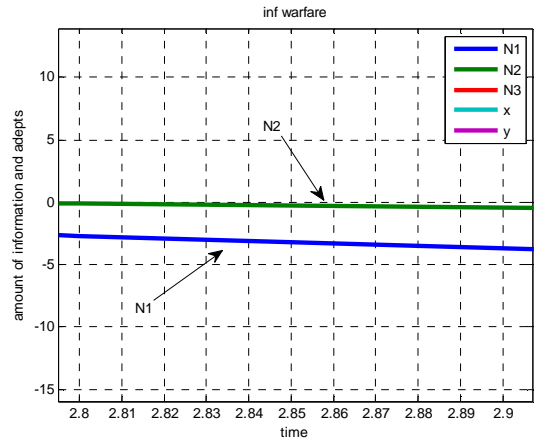


Fig. 9 Non-Permanent Information Warfare with Escalation of the first and attenuation of the second side, the end of the Information Warfare

Below are the m-files in the form of listing 3 using a computer experiment.

Listing 3. Non-Permanent Information Warfare with Escalation of the first and attenuation of the second side.

```
Gmcm_np_iw_oz.m
n0=[14 19 200 0.08 0.03];
[T,Y]=ode15s('oz_np_gmcmiw',[0
40],n0);plot(T,Y,'LineWidth',3)
title('inf warfare')
xlabel('time')
ylabel('amount of information and adepts')
legend('N1','N2','N3','x','y')
grid on
```

```
oz_np_gmcmiwit.m
%ode- right side of the system i
function dndt=oz_np_gmcmiwit(t,n)
dndt=zeros(5,1); ep=0.001;v1=0.001;
v2=0.005;ut=12; i1=155; i2=150;i3=200;
a1=0.9; a2=0.6; a3=0.07; a4=0.5;
a5=0.045; a6=0.8; a7=0.075; xp=7000;
yp=8000;b=.05;
dndt(1)=a1*log(1+log(1+t+ep))*(n(1)+v1*n
(4))*(1-n(1)/i1)-b*n(3);
dndt(2)=a2*(sin(pi*t/(2*ut))+ep)*(n(2)+v
2*n(5))*(1-n(2)/i2)-b*n(3);
dndt(3)=a3*log(1+log(1+t+ep))*(n(1)+n(2)
)*(1-n(3)/i3);
```

```

dndt(4)=n(1)*log(1+log(1+t+ep))*(a4+a5*n
(4))*(xp-n(4));
dndt(5)=n(2)*(sin(pi*t/(2*ut))+ep)*(a6+a
7*n(5))*(yp-n(5));
end

```

Now, consider the case when the second side attenuates its activity, and the first exacerbates. Let, for the first side

$$\alpha_1(t) = A_1 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right),$$

but for the second side we have

$$\alpha_2(t) = A_2 \arctg(t + \varepsilon).$$

Then our system take the form:

$$\begin{cases} \frac{d}{dt} N_1(t) = A_1 \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right) (N_1(t) + v_1 x(t)) \left(1 - \frac{N_1(t)}{I_1} \right) - \beta_1(t) N_3(t), \\ \frac{d}{dt} N_2(t) = A_2 \arctg(t + \varepsilon) (N_2(t) + v_2 y(t)) \left(1 - \frac{N_2(t)}{I_2} \right) - \beta_2(t) N_3(t), \\ \frac{d}{dt} N_3(t) = A_3 \arctg(t + \varepsilon) \times (N_1(t) + N_2(t)) \left(1 - \frac{N_3(t)}{I_3} \right), \\ \frac{dx(t)}{dt} = N_1(t) \left(\sin\left(\frac{\pi}{2\tau}t\right) + \varepsilon \right) (A_4 + A_5 x(t)) (x_p - x(t)), \\ \frac{dy(t)}{dt} = N_2(t) \arctg(t + \varepsilon) (A_6 + A_7 y(t)) (y_p - y(t)). \end{cases} \quad (6)$$

The study of the Cauchy problem (6), (2) showed that the Non-Permanent Information Warfare, which is described by the integrated model, is controllable, i.e., there are such values for γ , I_3 , at which the Information Warfare system from the state (2) goes into the state (3). However, for the controllability of system (6), (2), the peacekeeping side needs to make significant efforts, the value of peacekeeping activity γ should be increased by order of magnitude than it was for controllability in the previous case. For example, when $N_1(0)=10$, $N_2(0)=15$, $N_3(0)=19$, $x(0)=0.05$, $y(0)=0.03$, $A_1=0.9$, $A_2=0.6$, $A_3=0.07$, $A_4=0.5$, $A_5=0.045$, $A_6=0.8$, $A_7=0.075$, $\tau=5$, $\beta=0.05$, $\gamma(t)=A_3 \arctg(t + \varepsilon)$, $v_1=0.001$, $v_2=0.005$, $\varepsilon=0.001$, $\tau=12$, $I_1=155$, $I_2=150$, $I_3=250$, $x_p=7000$, $y_p=8000$, the second side does not stop the Information Attack, unlike the first side, see Fig. 9, but with increases in peacekeeping activity up to $\gamma(t)=A_3 \arctg(t + \varepsilon)$, $A_3=820.07$, and the technological

level of the third side up to $I_3=1750$, then both antagonistic sides complete the Information Attacks, the first side at the time $t^*=0.4749$, and the second at the time $t^{**}=0.6215$, see Fig. 10.

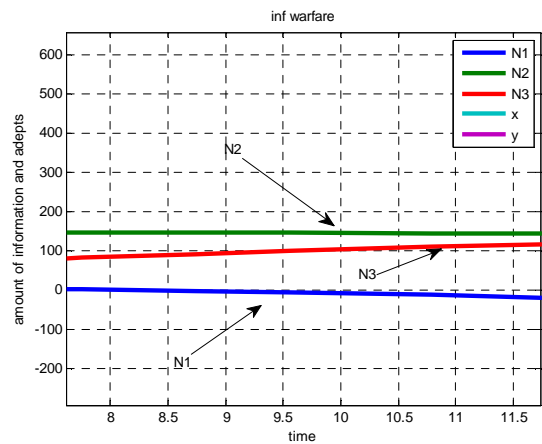


Fig. 10 Non-Permanent Information Warfare with the Escalation of the second and the attenuation of the first side. Only the first antagonistic side completes information attacks

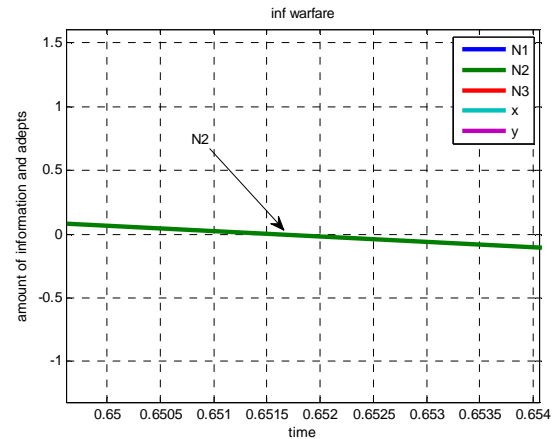


Fig. 11 Non-Permanent Information Warfare with the Escalation of the second and the attenuation of the first side. Both antagonistic party's complete information attacks.

Below are the m-files in the form of listing 4 using a computer experiment.

Listing 4. Non-Permanent Information Warfare with Escalation of the second and attenuation of the first side.

```

Gmcm_np_iw_zo.m
n0= [14 19 20 0.08 0.03];
[T, Y] =ode15s('zo_np_gmcmiw', [0 40],
n0); plot(T, Y,'LineWidth',3)
title('inf warfare')
xlabel('time')
ylabel('amount of information and
adepts')
legend('N1','N2','N3','x','y')
grid on

zo_np_gmcmiw
%ode- right side of the system i
function dndt=zo_np_gmcmiwit(t,n)
dndt=zeros(5,1); ep=0.001; v1=0.001;
v2=0.005; ut=12; i1=155; i2=150; i3=250;

```



```

a1=0.9;    a2=0.6;    a3=0.07;    a4=0.5;
a5=0.045; a6=0.8;    a7=0.075;    xp=7000;
yp=8000;  b=.05;
dndt(1)
=a1*(sin(pi*t/(2*ut))+ep)*(n(1)+v1*n(4))
*(1-n(1)/i1)-b*n(3);
dndt(2)
=a2*atan(t+ep)*(n(2)+v2*n(5))*(1-
n(2)/i2)-b*n(3);
dndt(3)    =a3*atan(t+ep)*(n(1)+n(2))*(1-
n(3)/i3);
dndt(4)
=n(1)*(sin(pi*t/(2*ut))+ep)*(a4+a5*n(4))
*(xp-n(4));
dndt(5)
=n(2)*atan(t+ep)*(a6+a7*n(5))*(yp-n(5));
end

```

IV. CONCLUSION

In the work, its Integrated Mathematical Model of Non-Permanent Information Warfare is built. Computer and mathematical methods are used to study the model problems of the Non-Permanent Information Warfare. For the peacekeeping side, recommendations were made to end the Information Warfare. The case of mutual attenuation and escalation of activity in different modes of antagonistic sides is considered. For a Non-Permanent Information Warfare with mutual attenuation, the system is controllable, and it is revealed that to end the Information Warfare the peacekeeping side does not require special efforts if there is no task of completing the confrontation at a certain point in time. For a Non-Permanent Information Warfare, with Escalation in different modes, the controllability of the system was established, and, taking into account the moment in time τ , tactics of the peacekeeping side to end the Information Warfare were developed. The data obtained for the integrated model of the Non-Permanent Information Warfare are in good agreement with the results obtained for the traditional model of the Information Warfare [1].

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